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EXPERIMENTAL RESULTS OF IMPULSIVE SYNCHRONIZATION BETWEEN TWO CHUA'S CIRCUITS

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Impulsive synchronization of chaotic dynamic systems has some important applications to chaotic secure communication and chaotic spread-spectrum communication systems. In this paper we present some experimental results on impulsive synchronization between two Chua's circuits. In our experiments, only one synchronizing impulse sequence is transmitted. The robustness of impulsive synchronization with respect to variations in the frequency and the width of impulses is studied. Experimental results show that robust impulsive synchronization can be achieved under noisy conditions and a 2% parameter mismatch between the driving system and the driven system. We also found that an amplified impulse sequence with a gain greater than unity can make the impulsive synchronization more robust. Moreover, we found that impulsive synchronization can be achieved with very narrow impulses.

1. Introduction

The concept of impulsive control and synchronization of chaotic systems was first reported in [Amritkar & Gupte, 1993; Stojanovski *et al.*, 1996]. After that, this problem was found closely connected to the theory of impulsive differential equations [Yang & Chua, 1997c]. A rigorous theory of the asymptotic stability of impulsive control and synchronization of autonomous chaotic systems was presented in [Yang & Chua, 1997b, 1997c]. Some applications of impulsive synchronization of chaotic systems to chaotic secure communication, and to chaotic digital code-division multiple access (CDMA) systems were presented in [Stojanovski *et al.*, 1996; Yang & Chua, 1997b, 1997c] and in [Yang & Chua, 1997a], respectively.

All of the above results are theoretical. Since impulsive control and synchronization involves changing the state variables of continuous dynamic systems rapidly, it is necessary to investigate the problems that may arise in the *practical* implementation of impulsive synchronization. In this letter, we present some experimental results of impulsive synchronization between two Chua's circuits [Chua, 1994].

2. Experimental Configurations

In this letter, we choose Chua's circuit [Chua, 1994] as the chaotic system which is defined by

$$\begin{cases} \dfrac{dv_1}{dt} = \dfrac{1}{C_1}[G(v_2 - v_1) - f(v_1)] \\ \dfrac{dv_2}{dt} = \dfrac{1}{C_2}[G(v_1 - v_2) + i_3] \\ \dfrac{di_3}{dt} = -\dfrac{1}{L}v_2 \end{cases} \tag{1}$$

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where $f(\cdot)$ is the nonlinear characteristic of Chua's diode, defined by

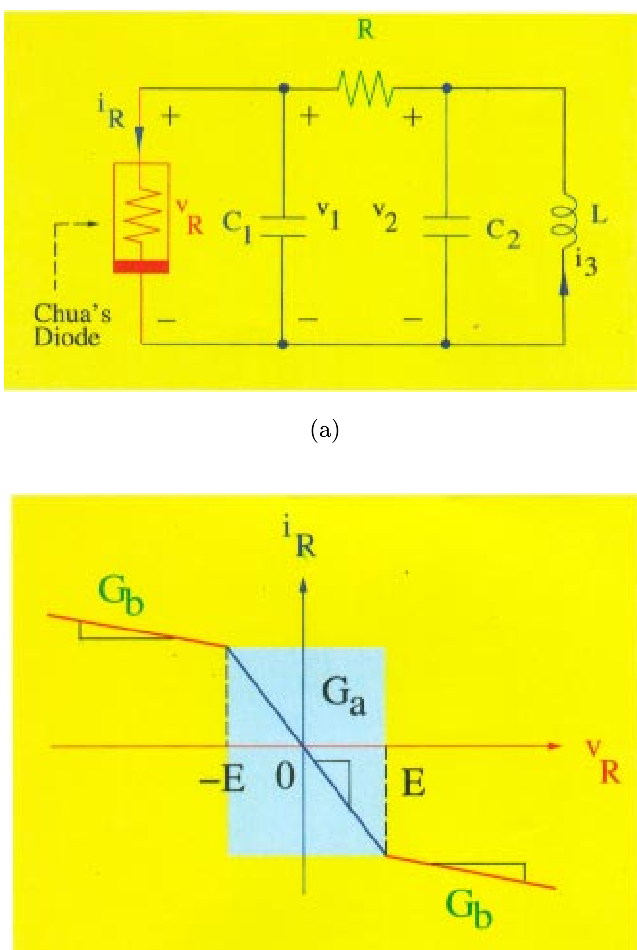
$$f(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)(|v_1 + E| - |v_1 - E|) \quad (2)$$

and E is the breakpoint voltage of Chua's diode. The corresponding circuit is shown in Fig. 1.

Although [Kennedy, 1992] had provided a physical implementation of Chua's circuit, for our present application, we have opted to use the circuit realization presented in [Dmitriev *et al.*, 1995], which we have found to be more robust for this application than the scheme given in [Kennedy, 1992].

The parameters chosen for this implementation are $C_1 = 5100$ pF, $C_2 = 47$ nF, $L = 18$ mH and $R = 1.65$ kΩ. The parameters for implementing Chua's diode are given by $E = 1.56$ V, $G_a = -0.757$ mS, and $G_b = -0.459$ mS. The Op Amps chosen in our implementation are type KR1401UD2A. With these parameters we obtain the Chua's double scroll strange attractor shown in the v_1–v_2 plane in Fig. 2, where the scale of the horizontal and the vertical axes is 0.2 V/div and 50 mV/div, respectively.

The block diagram of our impulsive synchronization using a single synchronizing impulse sequence is given in Fig. 3. Figure 3(a) shows the block diagram of impulsive synchronization via v_1 between two Chua's circuits. The switching signal $s(t)$ shown in Fig. 3(c) is used to switch on/off an electronic switching device (AD6 212 AKN). Whenever $s(t)$ assumes a high voltage level, the switch is on and a synchronizing impulse is transmitted from the driving system to the driven system. We call the width of the high voltage peaks as the *impulse width* w_i. The high voltage level of $s(t)$ is chosen to be greater than $5V$. The time interval between two consecutive impulses is called the *impulse period* $1/f_i$. Whenever $s(t)$ assumes a low voltage level, the switch is turned off and the two Chua's circuits are separated electronically. The low voltage level of $s(t)$ is chosen to be $0V$. Whenever $s(t)$ assumes a high voltage level, the two voltages $v_1(t)$ and $\tilde{v}_1(t)$ are connected by two voltage buffers, which force their output voltages to be equal to their input voltages, and hence $\tilde{v}_1(t)$ must tend rapidly to $v_1(t)$.



(a)

(b)

Fig. 1. (a) Chua's circuit. (b) Voltage-current characteristic of Chua's diode.

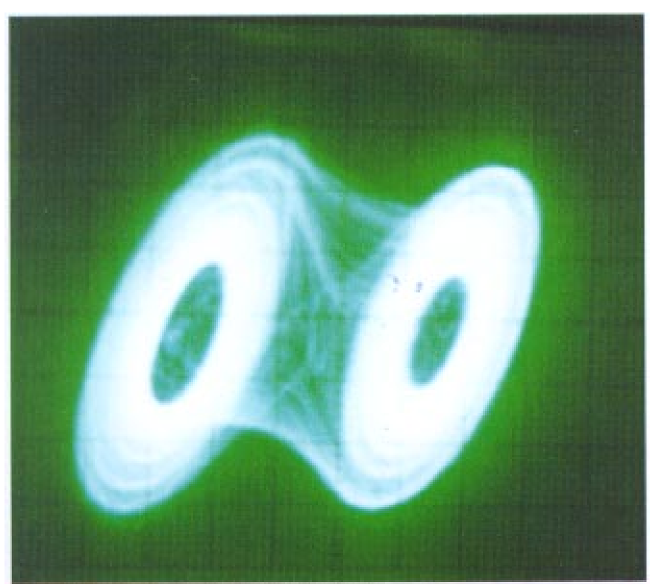
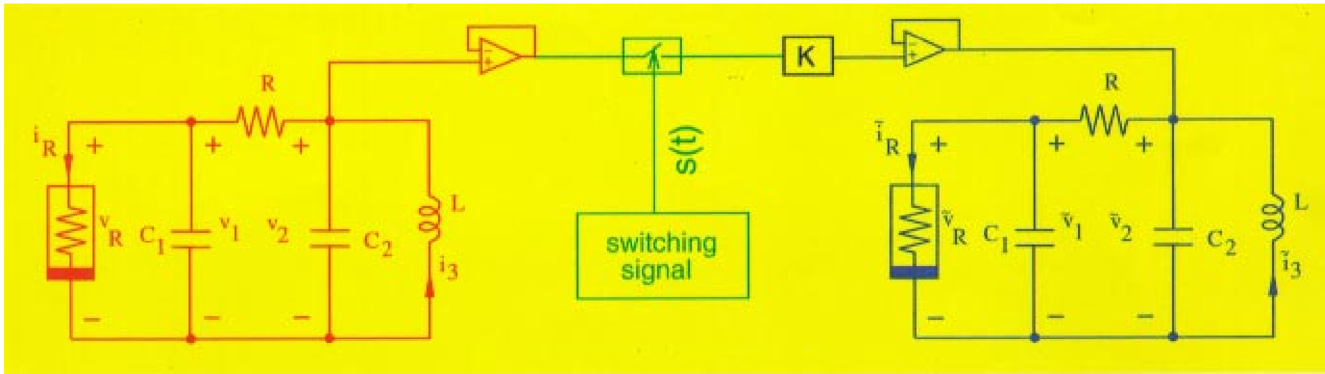
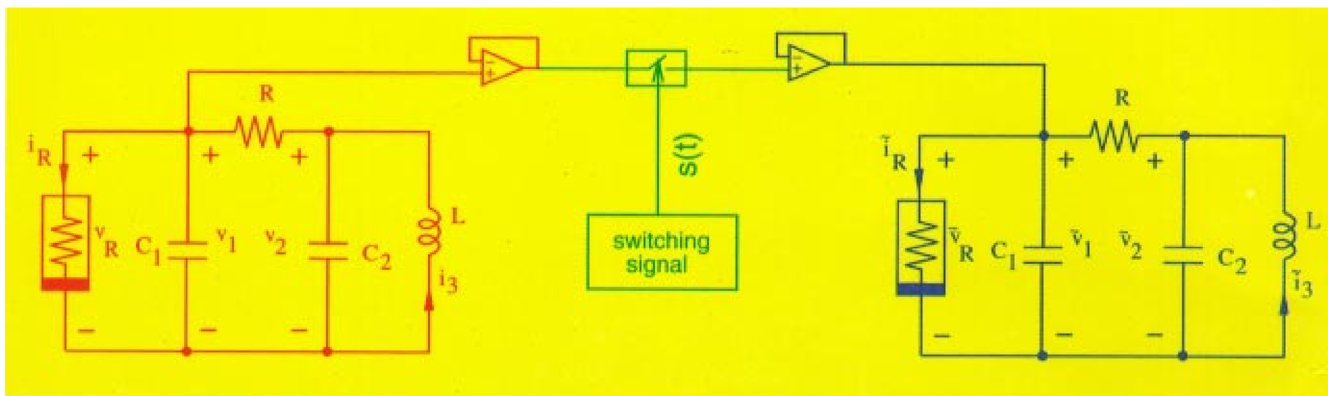


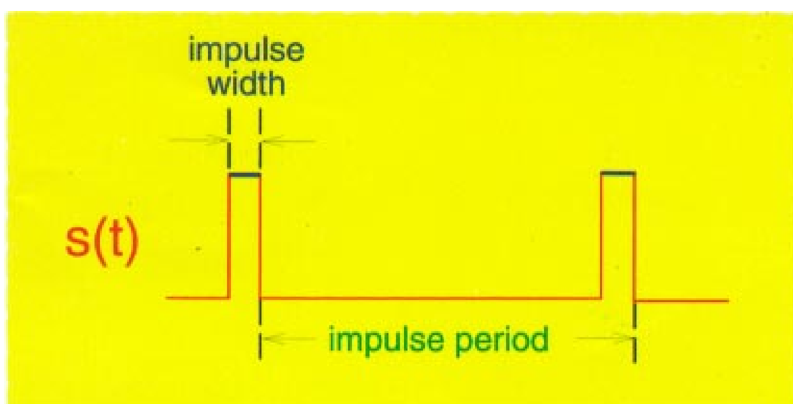
Fig. 2. The Chua's double scroll strange attractor observed from the Chua's circuit used in our experiments.



(a)



(b)



(c)

Fig. 3. The block diagram of impulsive synchronization between two Chua's circuits. (a) Impulsive synchronization via $v_1(t)$. (b) Impulsive synchronization via $v_2(t)$. (c) Illustration of switching signal $s(t)$.

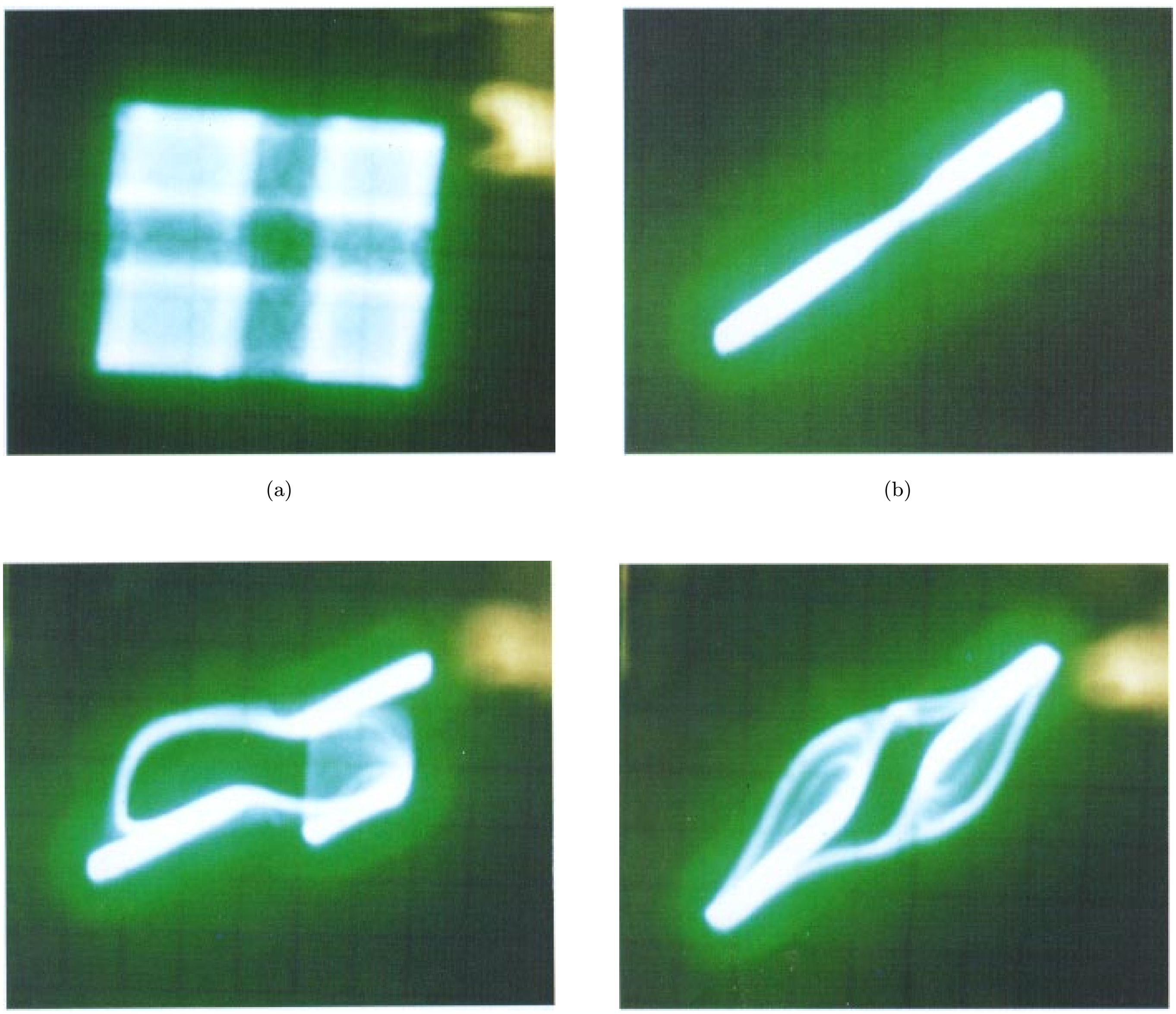
Figure 3(b) shows the block diagram for impulsive synchronization via v_2 between two Chua's circuits. In this case, the voltages of capacitors C_2 in both Chua's circuits are connected impulsively by two voltage buffers and an electronic switch. It is the same structure as that in Fig. 3(a) except that a channel gain K is introduced for increased generality.

3. Experimental Results

In this section, we present our experimental results of the configuration presented in Sec. 2. All pictures are taken directly from the screen of an oscilloscope.

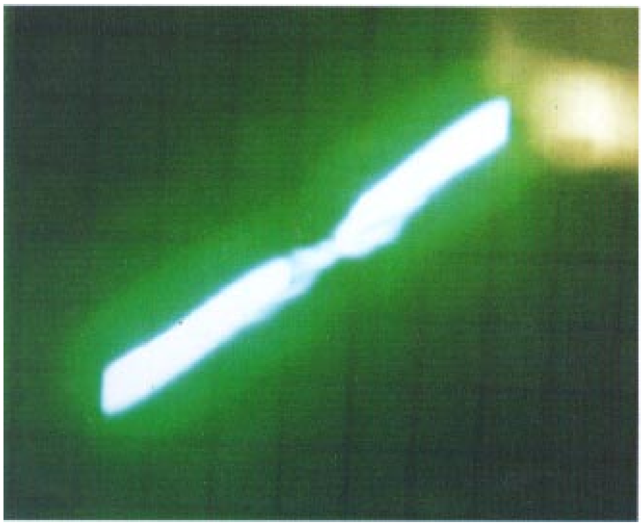
3.1. v_2 *is the transmitted signal*

We first study the cases when the voltage v_2 is the transmitted signal as shown in Fig. 3(b). The experimental results are shown in Fig. 4. Figure 4(a) shows, in the v_1–$\tilde{v}_1$ plane, that the two Chua's circuits are originally desynchronized if impulsive coupling is not applied. Figure 4(b) shows, in the v_1–$\tilde{v}_1$ plane, that the two Chua's circuits are synchronized if the impulsive coupling is applied. The parameters for the synchronization impulses are: Frequency of impulses is $f_i = 18$ kHz, width of impulses is $w_i = 32$ μs. Figure 4(c) shows, in the v_1–$\tilde{v}_1$ plane, that the two Chua's circuits



(a) (b) (c) (d)

Fig. 4. The experimental results of impulsive synchronization when v_2 is sampled as the synchronizing impulse sequence. In this figure, the scale of the oscilloscope is 0.2 V/div for both horizontal and vertical axes. (a) The driving system and the driven system are originally desynchronized. (b) Impulsive synchronization ($K = 1$). (c) Desynchronization due to narrow impulses ($K = 1$). (d) Desynchronization due to wide impulses ($K = 1$). (e) Impulsive synchronization with $K > 1$ and narrow impulses.



(e)

Fig. 4. (*Continued*)

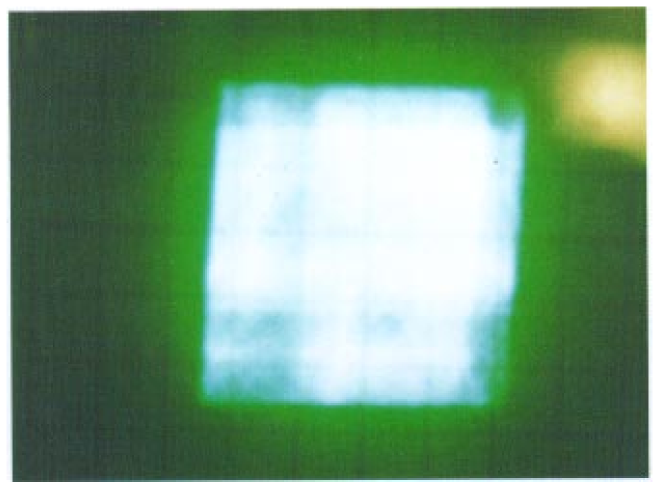
become desynchronized if we decrease the width of the impulse beyond a certain limit. The parameters for the synchronization impulses are: $f_i = 18$ kHz and $w_i = 24$ μs. Figure 4(d) shows, in the v_1–$\tilde{v}_1$ plane, that the two Chua's circuits become desynchronized if we increase the width of the impulse beyond a certain limit. The parameters for the synchronization impulses are: $f_i = 18$ kHz and $w_i = 46$ μs. Figure 4(e) shows, in the v_2–$\tilde{v}_2$ plane, that impulsive synchronization between two Chua's circuits can be maintained with narrow impulses if the amplitude of the impulses is amplified by a gain $K > 1$. In this case, the parameters for the synchronization impulses are: $f_i = 18$ kHz, $w_i = 12$ μs and $K = 2.67$.

3.2. *v_1 is the transmitted signal*

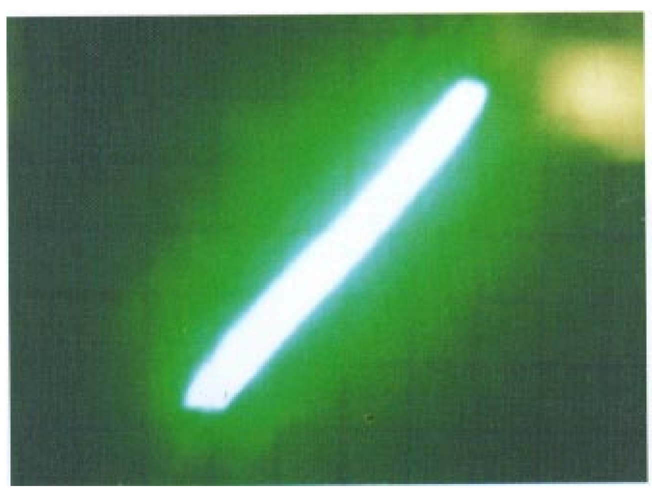
We study next the cases when the voltage v_1 is the transmitted signal. The experimental results are shown in Fig. 5. Figure 5(a) shows, in the v_2–$\tilde{v}_2$ plane, that the two Chua's circuits are originally desynchronized if impulsive coupling is not applied. Figure 5(b) shows, in the v_2–$\tilde{v}_2$ plane, that the two Chua's systems are synchronized if impulsive coupling is applied. The parameters for the synchronization impulses are: $f_i = 20$ kHz and $w_i = 32$ μs. Figure 5(c) shows, in the v_2–$\tilde{v}_2$ plane, that impulsive synchronization in this case is very robust using only *narrow* impulses, where the parameters of the synchronization impulses are: $f_i = 20$ kHz and $w_i = 8$ μs. Figure 5(d) shows the switching signal $s(t)$ used in Fig. 5(c).

4. Conclusions

In this letter we present two different schemes for implementing impulsive synchronization between two chaotic electronic circuits, namely, Chua's circuits, via a single synchronizing impulse sequence. We observed that the impulsive synchronization is robust in the presence of a 2% parameter mismatch between the driving and the driven systems under noisy conditions. However, the impulsive synchronization between two Chua's circuits via a

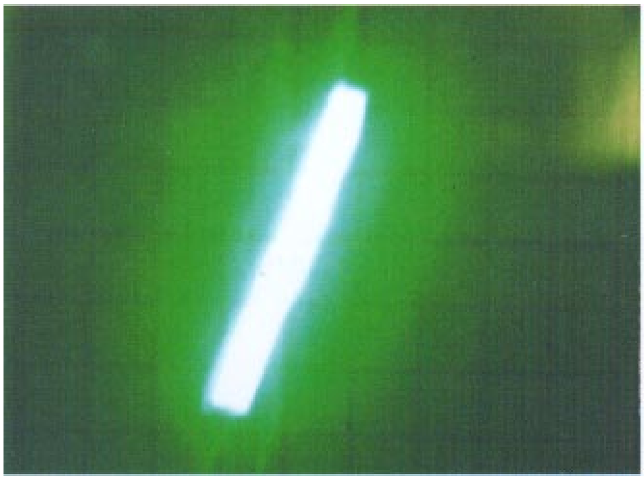


(a)

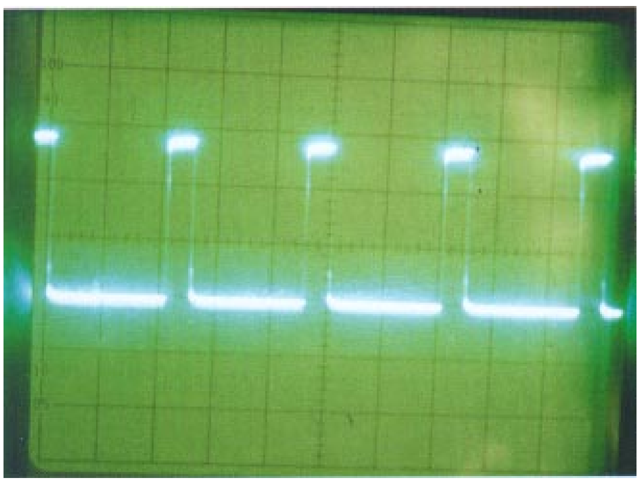


(b)

Fig. 5. The experimental results of impulsive synchronization when v_1 is sampled as the synchronizing impulse sequence. In this figure, the scale of the oscilloscope is 50 mV/div for both horizontal and vertical axes. (a) The driving system and the driven system are originally desynchronized. (b) Impulsive synchronization. (c) Impulsive synchronization with narrow impulses. (d) The switching signal $s(t)$ used in (c).



(c)



(d)

Fig. 5. (*Continued*)

single synchronizing impulse sequence is not asymptotically stable from a theoretical point of view. This is the reason why the frequency of the synchronizing impulse was chosen to around 10 kHz in our experiments.

Our experimental results show that the v_1-driving scheme is more robust than the v_2-driving scheme. This is because v_1 is directly connected to Chua's diode, which is the only nonlinearity in Chua's circuit.

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