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A scaling property of the fluctuation spectrum for an intermittency observed in the Chua circuit

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An intermittency experimentally observed in the Chua circuit is characterized by the fluctuation-spectrum theory. Near the intermittent transition point a scaling property is found for the fluctuation spectrum $\sigma(\alpha)$.

Recently, the fluctuation-spectrum theory has been developed in order to characterize the fluctuation of stochastic time series [1]. Some scaling properties of fluctuation spectra have been found near the intermittent transition point [2]. In experiment Fukushima et al. also showed scaling laws for an intermittency in a coupled chaos system [3]. It is important to investigate the scaling property for various types of intermittency observed in experimental systems. It is well known that chaos occurs in the Chua circuit [4]. An attractor in the Chua circuit is called double-scroll, which shows the rotation about two unstable fixed points like a Lorenz attractor. An intermittency can be observed with the transition from the single-scroll state to the double-scroll state. In the present Letter we investigate a scaling property of the fluctuation spectrum for an intermittency in the Chua circuit.

The circuit diagram is shown in fig. 1. The Chua circuit can be regarded as a nonlinear *LCR* oscillator which consists of two capacitors C_1 and C_2, an inductor L, a variable resistor R and a negative three-segment piecewise-linear resistor *NR*. See ref. [4] for the detailed structure. The values of the elements are the following: $C_1=0.0047$ μF, $C_2=0.047$ μF and $L=22$ mH. The value of R, which is a control parameter, can be varied from 0.0 to 2050.0 Ω. The voltage across C_1, V_{C_1}, is sampled as digital data by using an A/D converter. A typical intermittent time series of V_{C_1} is shown in fig. 2. This intermittency occurs due to switching between two states. Intervals between two successive switching events become long as the value of R approaches the critical value R_c. Varying the value of R, we take the time series of V_{C_1} near the intermittent transition point. In measurements the sampling time, which is chosen to be nearly equal to the characteristic period of oscillation, is set to 0.2 ms.

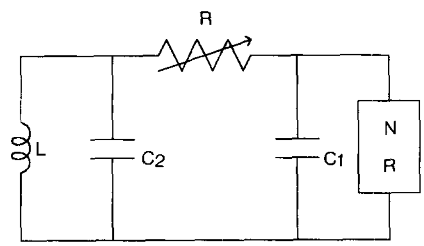

Fig. 1. Chua circuit diagram.

Now we briefly summarize the fluctuation-spectrum theory [2]. For the time series $\{u_i;\ i=1, 2, 3, \ldots\}$, we consider the local average of u_i,

$$\alpha_n = \frac{1}{n}\sum_{i=1}^{n} u_i \,. \tag{1}$$

The probability density $\rho_n(\alpha')$ that a value takes between α' and $\alpha'+\mathrm{d}\alpha'$ for large n is asymptotically written as

$$\rho_n(\alpha') \sim \exp[-\sigma(\alpha')n]\,, \tag{2}$$

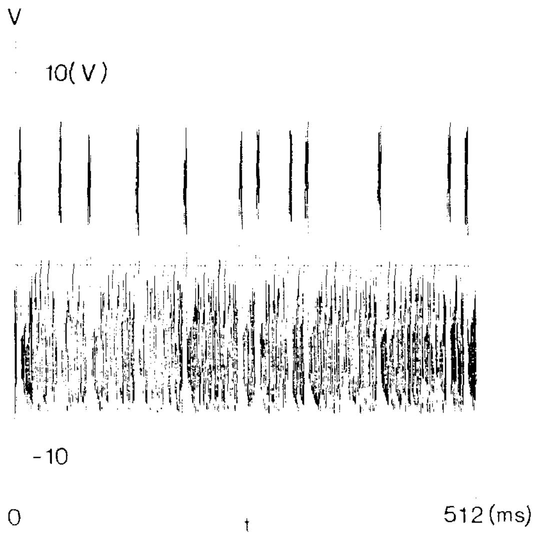

Fig. 2. Waveform of V_{C_1}. The value of R is 1980.0 Ω. The sampling time is 50 μs.

where $\sigma(\alpha')$ is called the fluctuation spectrum. A characteristic function is introduced in the following form,

$$\lambda_q = \frac{1}{q} \lim_{n\to\infty} \frac{1}{n} \log M_q(n) , \tag{3}$$

where

$$M_q(n) \equiv \langle \exp(qn\alpha_n) \rangle , \tag{4}$$

and $\langle \ \rangle$ is an ensemble average. The quantities of α and $\sigma(\alpha)$ are given by the Legendre transformation from q and λ_q,

$$\alpha = \frac{\mathrm{d}}{\mathrm{d}q} q\lambda_q , \tag{5}$$

$$\sigma(\alpha) = q^2 \frac{\mathrm{d}\lambda_q}{\mathrm{d}q} = q(\alpha - \lambda_q) . \tag{6}$$

If the time series $\{u_i\}$ takes two values, r_1 and r_2, the characteristic function λ_q can be rewritten as [4]

$$\lambda_q = \frac{1}{q} \log[p\exp(qr_1) + (1-p)\exp(qr_2)] , \tag{7}$$

where p is the probability that the u_i take the value r_1 and the parameters are given by

$$r_1 = \lambda_{-\infty} , \quad r_2 = \lambda_{\infty} . \tag{8}$$

For $q \sim 0$, λ_q can be written as

$$\lambda_q \sim \lambda_0 + Dq . \tag{9}$$

From the experimental data of V_{C_1} we construct a new time series $\{u_i\}$ by the following definition,

$$u_i = u_+ = \frac{a}{p} , \qquad V_{C_1} > 0.5 \text{ V} ,$$

$$= u_- = \frac{b-a}{1-p} , \quad V_{C_1} \leqslant 0.5 \text{ V} , \tag{10}$$

where a and b are parameters and p is the probability that the u_i take the value u_+. We choose $a = 0.05$ and $b = -0.05$. The number b is equal to the average of all data $\langle u_i \rangle$. We calculate the characteristic function λ_q and the fluctuation spectrum $\sigma(\alpha)$ for several data. In the calculation of λ_q we set $n = 1000$ and take the average over 32 ensembles so that a sufficient convergence of λ_q can be obtained. Results are shown in figs. 3a and 3b, where we choose $R = 1982.6$ Ω for A, $R = 1981.4$ Ω for B and $R = 1980.4$ Ω for C. Of the three data A is nearest to the critical point. We obtain the probability $p = 0.0064$ for A, $p = 0.0138$ for B and $p = 0.0364$ for C. The slope of λ_q near $q \sim 0$ becomes steep as the system approaches the critical point.

We consider the scaling property [5]. For $q \sim 0$, from eq. (9) λ_q is scaled in the form

$$\tilde{\lambda}_q \sim b + c_0 q / q^* , \tag{11}$$

where

$$q^* = c_0 / D \tag{12}$$

and the constant number c_0 is set to 2. In the limit of $p \to 0$ eq. (7) can be rewritten as

$$\tilde{\lambda}_q \sim \lambda_{-\infty} + \frac{A}{Q} (\mathrm{e}^{BQ} - 1) . \tag{13}$$

Comparing eqs. (9) and (13) we obtain

$$A = a^2 / 2c_0 , \quad B = 2c_0 / a , \quad Q = q / q^* , \tag{14}$$

where we use the relation

$$\lambda_{-\infty} = b - a . \tag{15}$$

The scaled fluctuation spectrum $\tilde{\sigma}(\alpha)$ and $\tilde{\alpha}$ are written in the same form as in eqs. (5) and (6),

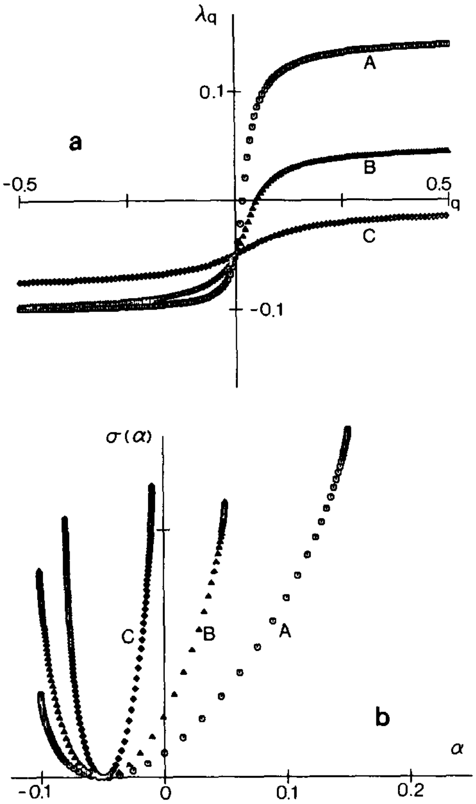

Fig. 3. (a) Characteristic function λ_q. (b) Fluctuation spectrum $\sigma(\alpha)$. The scale of the longitudinal axis is 0.0025. Data for (○) A, (△) B, (◇) C.

$$\tilde{\alpha}=\frac{\mathrm{d}}{\mathrm{d}Q}Q\tilde{\lambda}_q\,, \tag{16}$$

$$\tilde{\sigma}(\alpha)=Q^2\frac{\mathrm{d}\tilde{\lambda}_q}{\mathrm{d}Q}=Q(\tilde{\alpha}-\tilde{\lambda}_q)\,. \tag{17}$$

The scaled fluctuation spectra $\tilde{\sigma}(\alpha)$ are shown in fig. 4. The solid line in fig. 4 is the scaling function of $\tilde{\sigma}(\alpha)$ given by eq. (17). It is found that three $\tilde{\sigma}(\alpha)$'s assemble on a curve.

In this Letter we observed an intermittency in the Chua circuit. We calculated the characteristic function λ_q and the fluctuation spectrum $\sigma(\alpha)$ from the experimental data near the intermittent transition point by means of the fluctuation-spectrum theory.

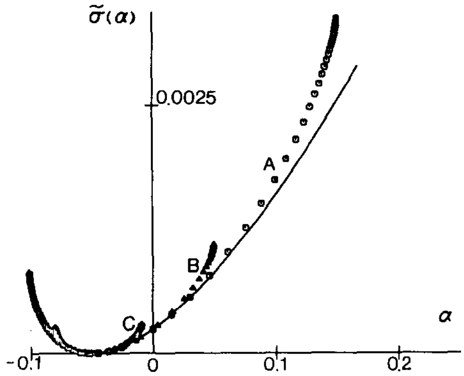

Fig. 4. The scaled fluctuation spectrum $\tilde{\sigma}(\alpha)$ together with the scaling function, eq. (17). Data for (○) A, (△) B, (◇) C.

λ_q and $\sigma(\alpha)$ are scaled and we found the scaling functions for $\tilde{\lambda}_q$ and $\tilde{\sigma}(\alpha)$. The scaling law holds in this intermittent transition similarly to the phase transition phenomena. We consider that q^* may be written by use of the deviation from the critical value ϵ $(=(R-R_c)/R_c)$ as

$$q^*\sim\epsilon^{\nu}\,. \tag{18}$$

However, we could not find this relation because exact determination of the critical value R_c was difficult in experiment. Finding this relation and determining the exponent ν are future problems.

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