Taming Chaos—Part I: Synchronization

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***Abstract*—The possibility of synchronization of systems inherently operating in a chaotic mode is analyzed. The Pecora–Carroll concept of synchronizable response subsystems and chaotic driving is described. Possibilities of synchronization using linear coupling of the chaotic systems are also considered. Potential applications of synchronized chaotic systems in signal processing are discussed and analyzed in examples using Chua's circuits.**

I. Introduction

DURING THE last decade, we observed a tremendous increase of interest in chaotic phenomena in a variety of physical systems (examples of what is commonly called "complex behavior" come from nuclear physics, laser optics, solid state physics, biology and medicine, socioeconomics, electrical engineering, mechanical and chemical engineering, and many other subject areas). In parallel with observations of complexity and chaos, there was an avalanche development in research allowing us to take measurements, analyze experimental data, quantify the behavior, and understand the underlying mechanisms. The results of this research can be found in thousands of conference and journal papers, books, and workshop and seminar presentations.

Despite this widespread interest, until very recently, the domain of chaos was considered an academic one, without deeper implications for real life applications. Chaos was considered rather an unwanted phenomenon, often hazardous for the operation of real physical systems, causing their malfunctioning and thus being a regime of operation to be avoided. In fact, the methods developed by scientists studying chaos, if used in applications at all, were used for designing chaos-free systems.

However, in real-life situations, we encounter chaos in *normal* system operation. Let us give two examples only: the first one, the dynamics of weather (climate), an enormous and extremely complex system exhibiting an abundance of dynamic behaviors on the macro scale; and the second one, the human brain, operating in chaotic mode on the micro scale [12], [22], [24].

Looking at these two types of chaotic systems, we wonder whether it is possible at all to find mechanisms for externally influencing their functioning (e.g., controlling rainfall) and, on the other hand, how one can explain and possibly use, for other purposes, the amazing ability of performing useful tasks like signal processing (in many cases, with irreproducible excellence) out of cooperation of a large number of chaotic subsystems (as in the case of the brain) [22], [24].

There are three basic problems to study:

1. How to influence or even better control the behavior of a system operating in a chaotic regime?
2. How extremely complex systems operating in chaotic mode "organize themselves" in performing useful tasks?
3. What are the underlying mechanisms enabling the interactions between subsystems, each operating in a chaotic mode, producing such useful behavior, and how can these mechanisms be used in building other systems of practical importance?

In this paper, I would like to give some answers to these kinds of questions—the basic answers for the simplest possible systems and their interconnections. In the first part, I describe the concept of synchronization of chaotic systems, which can be viewed as the simplest kind of useful cooperation of chaotic systems. I also describe possible applications of such synchronized chaotic systems. As we will see, synchronized chaotic systems can have some interesting applications in signal processing and communication. It is also believed that synchronization plays a crucial role in information processing in living organisms, and learning synchronization mechanisms in large interconnections (arrays) of chaotic systems could lead to even more exciting applications, e.g., in image or speech processing.

In the second part, I consider possibilities of controlling chaotic systems, i.e., influencing systems operating in a chaotic mode in such a way to produce a desired, prescribed type of behavior. Having good control algorithms for simple chaotic systems and very powerful computational tools, one could start to think about more serious real life applications like controlling chaotic vibrations in airplanes, turbulent flows of fluids in chemical reactors, and many others. I believe that many applications are still beyond our imagination.

Throughout this paper, I concentrate on examples coming from electrical and electronic engineering, in most examples using a now standard chaotic electronic circuit: the Chua's oscillator [7], [17].

I do not claim that the problems are solved, and there are algorithms ready to apply. Instead, I give an overview of exciting research activities carried out in laboratories around the world promising to soon take advantage of chaotic behavior. One should bear in mind that the algorithms and methods used are also applicable to other physical systems. Interested readers are referred to the listed literature.

II. Synchronization in Chaotic Systems

The possibility of two or more chaotic systems oscillating in a coherent, synchronized way is not an obvious one.

Manuscript received April 15, 1993; revised manuscript received June 7, 1993. This work was supported by University of Mining and Metallurgy grant 11.120.15. This paper was recommended by Guest Editor L. O. Chua.

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IEEE Log Number 9211608.

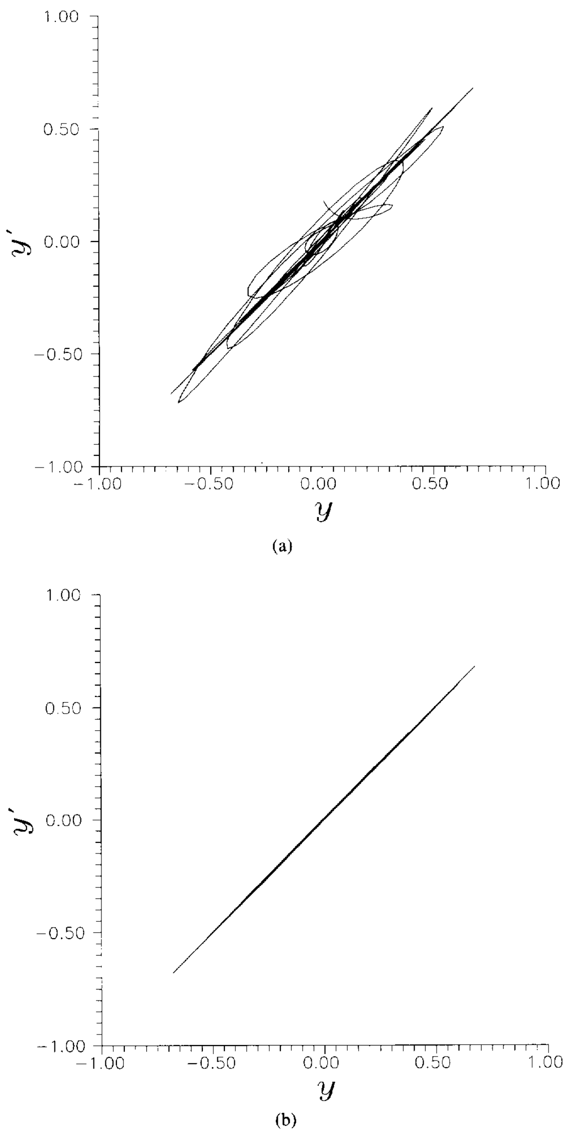

Fig. 1. Phase plot showing synchronization in the x-coupled Chua's circuits for $\delta_x = 0.5$ (horizontal axis: y, vertical axis: y'). In (a), a transient before reaching synchronization is clearly visible; when the transient dies out, nearly perfect synchronization is obtained in (b).

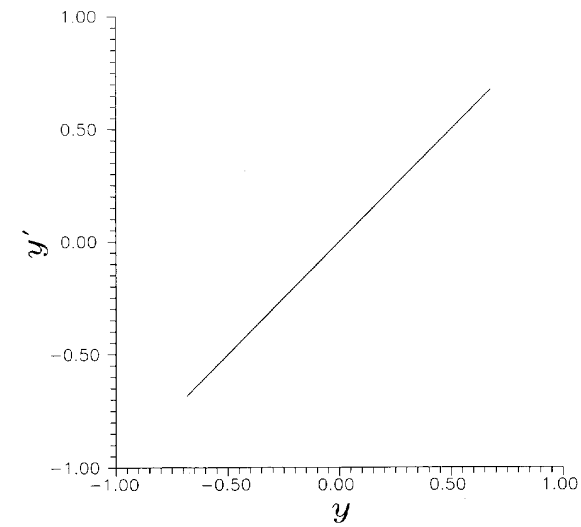

Fig. 2. Phase plot showing synchronization in the x-drive configuration of Chua's circuits (horizontal axis: y, vertical axis: y').

Considering one of the main features often mentioned in the definitions of chaotic behavior, namely, the property of sensitive dependence on initial conditions (instability in the Lyapunov sense), one could conclude that synchronization is not possible, because it is not possible in real systems either to reproduce exactly the same starting conditions or to match exactly the parameters of two systems. We are able to build "nearly" identical systems, but there is an inevitable technological mismatch and noise, impeding exact reproduction of all parameters. Thus, any even infinitesimal change of any parameter will eventually result in divergence of nearby starting orbits.

The Lyapunov stability concept of trajectories in a single system is not the proper one to analyze synchronization of two or more systems. In such a case, one should ask what are the conditions that imply the convergence of trajectories of the two systems rather than consider the stability of each one alone. In other words, having two (or more) nonlinear systems ($N \geq 2$):

$$\dot{x}_i = f_i(x_i), \qquad x \in R^n, \qquad 1 \leq i \leq N \tag{1}$$

we would like to find conditions under which their solutions will converge to each other, i.e.:

$$\lim_{t\to\infty}(x_i - x_j) = 0, \qquad i \neq j \tag{2}$$

There is no general answer to this problem. Below I describe some concepts for obtaining coherent (synchronous) operation of chaotic systems.

2.1. Linear Coupling

The simplest possibility considered in several papers [8], [19] is the linear coupling of the two systems we would like to synchronize:

$$\begin{aligned} \dot{x} &= f_1(x) \\ \dot{y} &= f_2(y) + \Delta(x - y) \end{aligned} \tag{3}$$

where $x, y \in R^n$, $\Delta = \operatorname{diag}[\delta_1, \cdots, \delta_n]^T$.

The synchronization problem is formulated as follows: Find Δ such that $y(t) \to x(t)$ for $t \to \infty$ (i.e., the solution $y(t)$ will synchronize with the signal $x(t)$).

This kind of linear coupling has been used for some particular types of systems. (The example of Chua's circuit is described below.) Kočarev *et al.* [20] have given some theorems concerning the convergence of solutions x and y. The following are the most interesting results:

Case $f_1 = f_2$:

Theorem 1: If $f_1 = f_2$ and $|x(t=0) - y(t=0)|$ [20] is sufficiently small, then there exist finite values $\bar{\delta}_i$, with $i = 1, 2, \cdots, n$, such that for $\delta_i > \bar{\delta}_i$ $y(t)$, approaches the goal $x(t)$.

Case $f_1 \neq f_2$: For simplicity, I assume that $\delta_i = k$, for all $i = 1, 2, \cdots, n$. Equation(3) can be rewritten as:

$$\begin{aligned} \dot{x} &= f_1(x) \\ \dot{y} &= f_2(y) + k(x-y) \end{aligned} \tag{4}$$

where k is a real nonnegative parameter.

Theorem 2: For $\epsilon = \delta^{-1}$ and sufficiently small $|x(t=0)| + |y(t=0)|$, there exists t_0 such that $y(t)$ converges uniformly to $x(t)$ as $\epsilon \to 0^+$ on all closed subsets of $t_0 < t < \infty$.

These two theorems give some very general conditions for synchronization. Theorem 2 not only enables us to treat the case when there is a parameter mismatch, but the systems are nearly identical, but can also be applied for obtaining synchronization of systems with quite different dynamics! One should note, however, that the problem of choosing the initial conditions of the two systems is of particular importance: the theorems do not tell us much about the regions of convergence.

2.2. Pecora–Carroll Drive-Response Concept

Thus far, the most effective and widely studied approach is due to Pecora and Carroll [4], [5], [6], [27], [28], who proposed a solution to a class of synchronization problems.

They considered an n-dimensional autonomous system governed by a state equation of the form:

$$\frac{dx}{dt} = f(x(t)) \tag{5}$$

Divide the system into two parts in an arbitrary way, thus dividing the state vector into $x = \begin{bmatrix} x_D \\ x_R \end{bmatrix}$. The D part is referred to as the driving subsystem, and the R part is referred to as the response subsystem respectively. Then:

$$\begin{aligned} \dot{x}_D &= g(x_D, x_R) \\ \dot{x}_R &= h(x_D, x_R) \end{aligned} \tag{6}$$

where $x_D = [x_1, \cdots, x_m]^T$, $x_R = [x_{m+1}, \cdots, x_n]^T$, $g = [f_1(x), \cdots, f_m(x)]^T$, $h = [f_{m+1}(x), \cdots, f_n(x)]^T$.

Pecora and Carroll suggested building an identical copy of the response subsystem and drive it with the x_D variables coming from the original system. In such a situation, we obtain the following equations:

$$\begin{aligned} \dot{x}_D &= g(x_D, x_R) \\ \dot{x}_R &= h(x_D, x_R) \\ \dot{x}'_R &= h(x_D, x'_R) \end{aligned} \tag{7}$$

Let us next examine the difference $\Delta x_R = x'_R - x_R$. The subsystem components x_R and x'_R will asymptotically approach each other (synchronize) if $\Delta x_R \to 0$ for $t \to \infty$. In the limit, this leads to the variational equations for the response subsystem:

$$\frac{d\Delta x_R}{dt} = D_{x_R} h(x_D(t), x_R(t))\, \Delta x_R + o((\Delta x_R)^2) \tag{8}$$

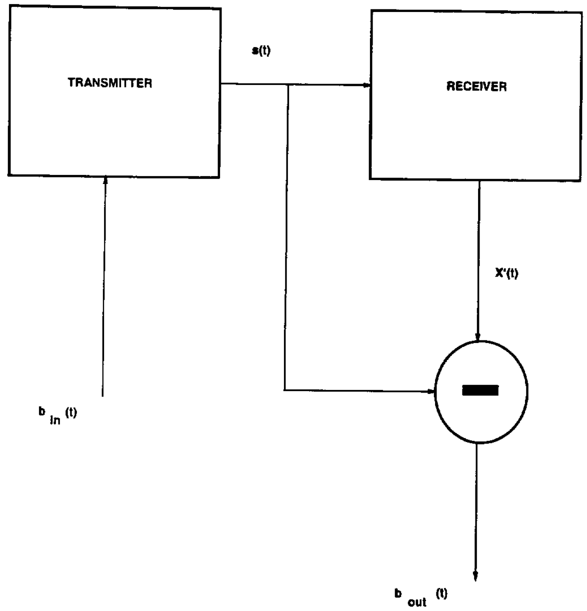

Fig. 3. Block diagram of the transmission system using chaos switching technique.

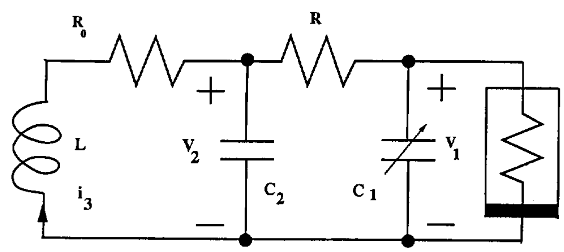

Fig. 4. Transmitter circuit: Chua's circuit with a variable capacitor.

where D_{x_R} denotes the Jacobian of the response subsystem with respect to x_R only. The behavior of the solutions of this system depends on the so-called conditional (depending on x_D) Lyapunov exponents measuring the average convergence or divergence rate of nearby points in the state space.

Pecora and Carroll proposed the following necessary condition for chaotic synchronization.

Theorem: The subsystems x_R and x'_R will synchronize only if the conditional Lyapunov exponents are all negative.

This methodology has been successfully applied to obtain chaos synchronization in coupled Lorenz systems [25], Rössler systems [27], and the hysteretic circuit [4]. Finally, Pecora and Carroll proposed a specific laboratory circuit for studying synchronization phenomena [5]. Interesting results have been also obtained for coupled Chua's circuits. Some of these are presented in the following section.

2.3. Synchronization Examples: Chua's Circuit

Several experimental studies have been carried out to show chaotic synchronization [8]. Extensive experimental studies were done using Kennedy's [17] implementations of Chua's

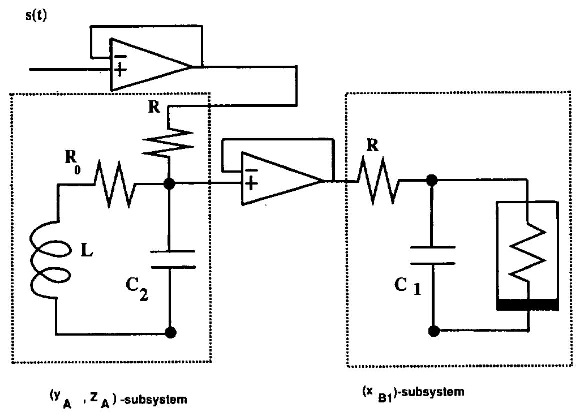

Fig. 5. Receiver circuit built by using two partial Chua's circuits.

circuits. It seems that experiments using Chua's circuits are the easiest to perform, and some of the details are given below.

Coupled Chua's Circuits: Let us consider a linear coupling of two Chua's circuits:

$$\begin{aligned} \dot{x} &= \alpha(y - x - f(x)) + \delta_x(x' - x) \\ \dot{y} &= x - y - z + \delta_y(y' - y) \\ \dot{z} &= -\beta y + \delta_z(z' - z) \\ \dot{x}' &= \alpha(y' - x' - f(x')) + \delta_x(x - x') \\ \dot{y}' &= x' - y' - z' + \delta_y(y - y') \\ \dot{z}' &= -\beta y' + \delta_z(z - z') \end{aligned} \tag{9}$$

Chua *et al.* [8] have shown that two coupled Chua's circuits characterized by $\alpha = 10.0$, $\beta = 14.87$, $a = -1.27$, $b = -0.68$ will synchronize; i.e., the solutions of the two systems will approach each other asymptotically for the following sets of parameters δ:

1. $\delta_x > 0.5$, $\delta_y = \delta_z = 0$,
2. $\delta_y > 5.5$, $\delta_x = \delta_z = 0$,
3. $2 > \delta_z > 0.7$, $\delta_x = \delta_y = 0$

In the laboratory experiments, the x-coupling can be realized by inserting a R_x ($\delta_x = C_2R/C_1R_x$) resistor between the + terminals of the nonlinear resistors. The y-coupling can be realized by connecting an R_y ($\delta_y = R/R_y$) resistor between the + terminals of C_2 Capacitors.

Fig. 1 shows typical phase plots confirming synchronization of two chaotic Chua's circuits in the case of linear coupling. Transient behavior before reaching synchronization is clearly visible in Fig. 1(a). After removing the transient, nearly ideal synchronization is obtained (Fig. 1(b)).

Synchronization of Chua's Circuits Using the Drive-Response Concept: Considering the Pecora–Carroll approach, it has been confirmed [8] that in two configurations: x-drive configuration for which the state equations read:

$$\begin{aligned} \dot{x} &= \alpha(y - x - f(x)) \\ \dot{y} &= x - y - z \\ \dot{z} &= -\beta y \\ \dot{y}' &= x - y' - z' \\ \dot{z}' &= -\beta y' \end{aligned} \tag{10}$$

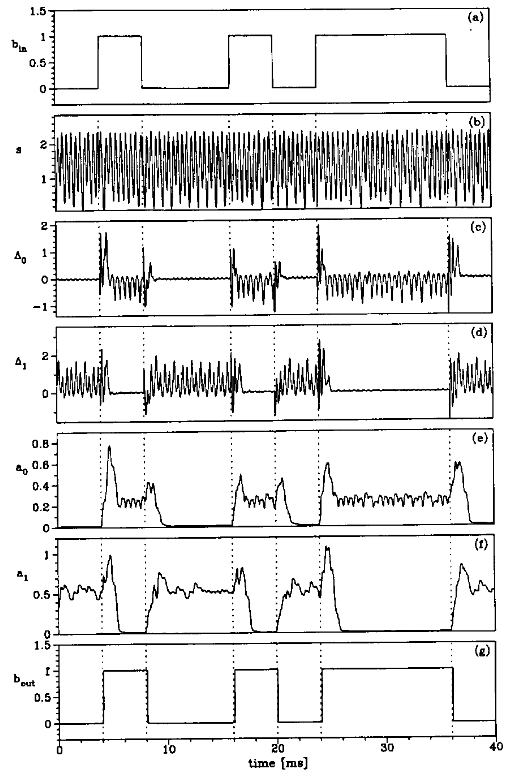

Fig. 6. Transmission of digital signals via parameter modulation (chaos switching) technique. 0 and 1 states are coded by two different chaotic attractors. (a) binary input signal b_{in}. (b) transmitted signal $s(t)$. (c) response Δ_0. (d) response Δ_1. (e) 40-point moving average of Δ_0. (f) 40-point moving average of Δ_1. (g) output binary signal b_{out}.

and the conditional Lyapunov exponents are $(-0.05, -0.05)$. Fig. 2 shows a typical phase plot confirming excellent synchronization of chaotic trajectories in this case. y-drive configuration for which the state equations become:

$$\begin{aligned} \dot{x} &= \alpha(y - x - f(x)) \\ \dot{y} &= x - y - z \\ \dot{z} &= -\beta y \\ \dot{x}' &= \alpha(y - x' - f(x')) \\ \dot{z}' &= -\beta y' \end{aligned} \tag{11}$$

and the conditional Lyapunov exponents were found $(-2.5, 0)$.

In the z-drive configuration, the subsystems do not synchronize; one of the conditional Lyapunov exponents was found to be positive.

III. Possible Applications

The earliest attempts to use random signals in secure communications date back to 1926, when Vernam published his paper [35]. (For the topic of cryptography, see the special issue of Proceedings of the IEEE [36].[1]

[1] Vernam proposed to use binary alphabet and the key only one time, i.e., to code each bit of the text with a new randomly chosen bit of the key. The coding principles were not changed since Caesar's times. Each of the letters of the text to be coded, x, has to be replaced by a symbol y obtained via chosen "modulo" summation with a secret key z, i.e., $y = x \oplus z$.

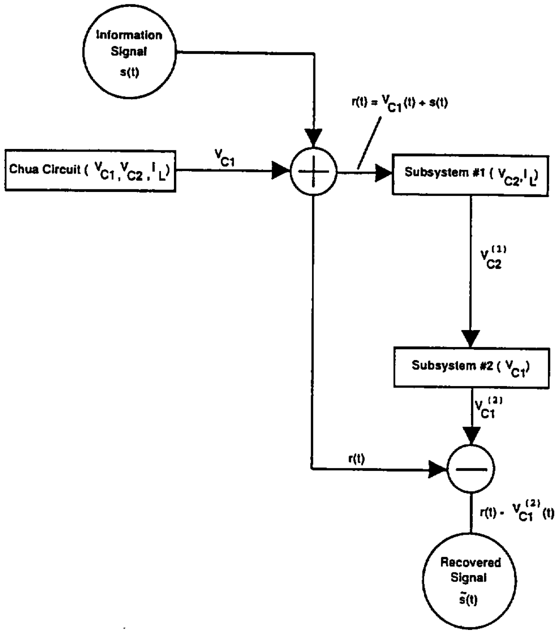

Fig. 7. Block diagram of secure communication system employing the masking principle.

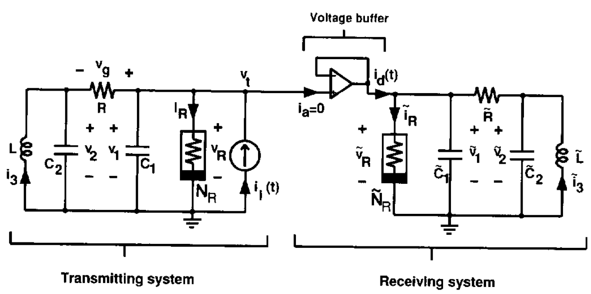

Fig. 8. Schematic diagram of the chaos modulation communication system employing two self-synchronizing Chua's circuits.

These ideas were explored again in the context of chaotic signals and used nearly simultaneously by independent research groups at the ONR ([6], [29]), University of California at Berkeley ([13], [19], [20], [26]), the Massachusetts Institute of Technology ([25]), and a joint from the Swiss Federal Institute of Technology and University College Dublin ([10], [18]).

It is the group of Prof. Leon O. Chua who published the first real circuit implementation and test results proving that the ideas of using chaotic signals and synchronized chaotic circuits in communication problems is not only useful but technically feasible, offering possibly competitive solutions to secure communication problems.

There is also an interesting alternative approach based on the information theoretic formalism of chaos reported recently in [14]; however, no implementations have been reported so far.

We will see in the simple examples that a chaos-producing system (Chua's circuit) can be used as the enciphering key. This key is fully identified by the actual circuit parameters.

3.1. Chaotic Switching

The simplest idea of how chaotic systems can be used in data transmission is the parameter modulation or chaotic switching [25]. The basic idea is to encode the binary signal in terms of different attractors existing for different system parameter values in the system (e.g., the 1- corresponds to parameter value μ_1, and further, chaotic attractor $\mathcal{A}_1$, 0- corresponds to parameter value μ_2 and a chaotic attractor $\mathcal{A}_2$). The chaotic system behavior is switched between $\mathcal{A}_1$ and $\mathcal{A}_2$; thus, the time response of the system is modulated by parameter changes.

The usefulness of this simple idea has been demonstrated by Parlitz *et al.* [26]. The block diagram of the proposed transmission system is shown in Fig. 3. Chua's circuit has been used as a source of chaotic signals Fig. 4. In the simulation experiments, the parameters $R = 1001\ \Omega$, $R_0 = 20\ \Omega$, $G_a = -1.139$ mS, $G_b = -0.711$ mS, $B_p = 1$ V were fixed, while the other parameters were switched between $L = 12$ mH, $C_1 = 17$ nF and $C_2 = 178$ nF for $b_{in} = 1$ (first parameter set) and $L = 13.3$ mH, $C_1 = 18.8$ nF, and $C_2 = 197$ nF for $b_{in} = 0$ (second parameter set), depending on the binary input signal $b_{in}(t)$. In both cases, the system possesses qualitatively similar attractors. The voltage across the capacitor C_1 has been chosen as the signal to be transmitted $s(t)$. The transmitted signal in both cases is chaotic and thus broadband. Following the Pecora–Carroll principles, the receiver (Fig. 5) is built as a copy of a part of the transmitter Chua's circuit (y_A, z_A—subsystem #1) with the first set of parameters. To determine whether there is indeed synchronization, reference signals have to be generated by using the known variables $x = v_{C_1}$, y_A, z_A. For this purpose, the subsystem #2 has been added (reproducing the variable x_{B1}). This system synchronizes only for one of the transmitted states, i.e., when the quantity $\Delta_0 = x - x_{B1} = 0$. Second, Chua's circuit (built with the second set of parameters) can also be added to reproduce the variable x_{B2} and synchronize with the chaotic signal in the second state only ($\Delta_1 = x - x_{B2} = 0$). The use of two chaotic signals with mutually exclusive synchronization properties improves the reliability of the system.

Fig. 6 presents the waveforms obtained in simulations. The waveforms represent, respectively, the following: (a) binary input signal b_{in}, (b) transmitted signal $s(t)$, (c) response Δ_0, (d) response Δ_1, (e) 40-point moving average of Δ_0, (f) 40-point moving average of Δ_1, and (g) output binary signal b_{out}, $\epsilon = 0.1$. b_{out} was derived by using the rule:

$$b_{out} = \begin{cases} 0,\ b_{old} = 0 & \text{for} \quad a_0 < \epsilon,\ \alpha_1 > \epsilon \\ 1,\ b_{old} = 1 & \text{for} \quad a_0 > \epsilon,\ a_1 < \epsilon \\ b_{old} & \text{for} \quad a_0 < \epsilon,\ a_1 < \epsilon \\ 1 - b_{old} & \text{for} \quad a_0 > \epsilon,\ a_1 > \epsilon \end{cases} \tag{12}$$

The resulting digital signal b_{out} agrees up to a small time delay with the original input signal n_{in}. Parlitz *et al.*, in their study [26], for the first time presented results of laboratory experiments demonstrating the applicability of the proposed method and showing that secure communication using chaotic switching is possible and might lead to new developments in communication techniques.

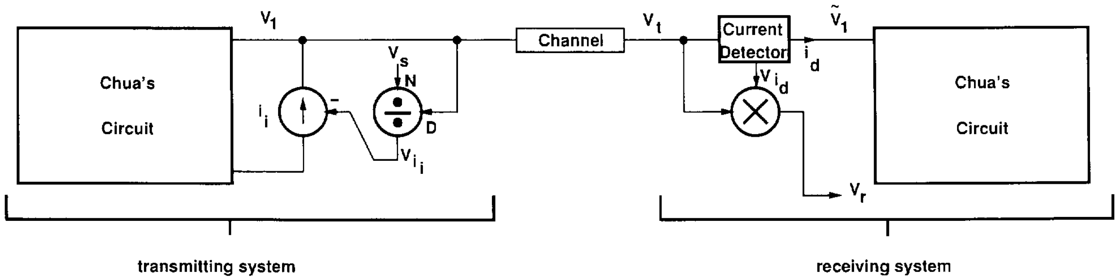

Fig. 9. Block diagram of the laboratory chaos modulation system. In the divider, N stands for the numerator and D stands for the denominator.

3.2. Chaotic Masking: Secure Communication

Another possibility of using chaotic signals for secure communication is using them for masking the information-carrying signal. This idea has been described by Oppenheim *et al.* [25] and Kocarev *et al.* [19]. The information-carrying signal is simply added to the masking chaotic signal. In [19], the authors report on using Pecora–Carroll approach and building an experimental set up for secure signal transmission based on the masking principle. Again, Chua's circuit has been used as a universal chaotic building block. The diagram of the proposed system is shown in Fig. 7. It contains a Chua's circuit in the transmitter part and two partial Chua's circuits in the receiver part. The receiver has exactly the same structure as in the case of the previous example (Fig. 5). The first subcircuit serves as a decoding key and synchronizes only when exactly matched with the transmitter circuit, thus reproducing the v_{C_2} signal. The second subcircuit is used for restitution of the missing variable v_{C_1} needed for recovering the information signal by simple subtraction as shown in the block diagram. In all laboratory tests in a real circuit implementation, it has been confirmed that a chaotic signal can be used as a masking signal and that it is possible to decode such a signal successfully by using Pecora–Carroll synchronization concept.

3.3. Chaotic Modulation: Spread-Spectrum Transmission

The most complex issue offered for secure communication has been described in a recent paper by Halle *et al.* [13]. The proposed idea is to multiply the information signal by a broad-spectrum, noiselike chaotic signal.

The transmission system shown in Fig. 8 uses two Chua's circuits. In the transmitting system, a current signal $i_i(t)$ is injected into the circuit and modifies the voltage across the capacitor C_1. This current signal depends on the input information $v_s(t)$ to be transmitted $i_i(t) = c(v_s(t))$ (where c is an invertible coding function). The detected current signal i_d is then decoded through $v_r(t) = c^{-1}(i_d(t))$. For proper operation, it is necessary that $v_r(t) \approx v_s(t)$. The coding function c should be chosen in such a way that during transmitter operation for all $v_s(t)$, the transmitted signal remains chaotic and looks the same. The voltage across the capacitor C_1 is transmitted through the channel to the receiver circuit and is used as a forcing voltage on the second Chua's circuit capacitor $\tilde{C}_1$.

Assuming that all circuit components of the transmitter and receiver are matched exactly, and inserting a voltage buffer to separate the two subsystems, we have $\tilde{v}_1 = v_1$. Using this condition and subtracting the circuit equations describing the dynamics of each of the Chua's circuits, one obtains

$$\begin{aligned} C_2\frac{d(v_2-\tilde{v}_2)}{dt} &= -\frac{1}{R}(v_2-\tilde{v}_2)) + (i_3-\tilde{i}_3)) \\ L\frac{d(i_3-\tilde{i}_3)}{dt} &= -(v_2-\tilde{v}_2) \end{aligned} \tag{13}$$

For R, L, $C_2 > 0$, we have [13] $(v_2(t) - \tilde{v}_2(t)) \to 0$ for $t \to \infty$. This implies also $i_d(t) \to i_i(t)$ and $v_r(t) \to v_s(t)$ for $t \to \infty$.

This means that the current flowing into the second Chua's circuit must equal (possibly after some transient) the current injected into the first Chua's circuit.

Halle *et al.* [13] describe the laboratory implementation of such a transmission system based on two synchronized Chua's circuits with the division operation $c(v_s(t)) = v_s(t)/v_1(t)$ chosen as the coding function and multiplication operation $v_r = c(v_s(t))v_1$ as the decoding one. The block diagram of the implemented system is shown in Fig. 9. This diagram could serve as a general principle of a transmission system using chaotic modulation.

The results presented in [13] demonstrate the feasibility of using self-synchronizing circuits (and in particular Chua's circuits) to implement spread-spectrum communication systems. For the full account of the experimental results, see the original paper [13]. I would like to stress here that this kind of chaotic signal modulation offers several advantages over the parameter modulation or simple masking techniques. First, the whole range of the chaotic signal spectrum is used for hiding the information. Second, the sensitivity to parameter variation is increased, thus offering increased security.

IV. Conclusions

The synchronization principles described in the previous sections enable us to build chaotic systems operating coherently and to use them to solve real communication problems. Understanding synchronization phenomena of simple interconnections of chaotic oscillators enabled several interesting developments such as switching, masking, and modulation using chaotic signals and could also serve as a basis for further studies.

There already exists a widespread interest in studies of higher-dimensional coupled chaotic systems in particular arrays of chaotic oscillators [1], [2], [3], [9], [11], [15], [16], [23], [30], [31], [34]. These types of systems are important as models of biological and physical systems and also from the information-processing point of view, offering possible engineering applications (e.g., [31]). One can expect a rapid development of research in this area.

ACKNOWLEDGMENT

Many discussion is with L. Kočarev, H. Dedieu, K. Halle, and M. P. Kennedy are greatly appreciated.

REFERENCES

[1] V.S. Afraimovich, V. Nekorkin, G.V. Osipov, and V.D. Shalfeev, "Stability, structure and chaos in nonlinear synchronisation arrays," in *IAP* (A. V. Gaponov-Grekhov and M. I. Rabinovich, Eds.), USSR Academy of Sciences, 1989 (in Russian).

[2] V. S. Anishchenko, T. E. Vadivasova, D. E. Postnov, and M. A. Safonova, "Synchronization of chaos," *Int. J. Bifurcation Chaos*, vol. 2, pp. 633–644, 1992.

[3] V.N. Belykh, N.N. Verichev, L. Kočarev, and L.O. Chua, "On chaotic synchronisation in a linear array of Chua's circuits," Memo. no. UCB/ERL M93/11, Jan. 29, 1993.

[4] T. L. Carroll and L. M. Pecora, "Synchronizing chaotic circuits," *IEEE Trans. Circuits Syst.*, vol. 38, pp. 453–456, 1991.

[5] ——, "A circuit for studying the synchronisation of chaotic systems," *Int. J. Bifurcation Chaos*, vol. 2, pp. 659–667, 1992.

[6] ——, "Cascading and controlling synchronized chaotic systems," Naval Res. Lab., 1992.

[7] L. O. Chua, "The genesis of Chua's circuit," *Archiv für Elektronik und Übertragungstechnik*, vol. 46, pp. 250–257, 1992.

[8] L. O. Chua, L. Kočarev, K. Eckert, and M. Itoh, "Experimental chaos synchronisation in Chua's circuit," *Int. J. Bifurcation Chaos*, vol. 2, pp. 705–708, 1992.

[9] J. P. Crutchfield, "Spatio-temporal complexity in nonlinear image processing," *IEEE Trans. Circuits Syst.*, vol. 35, pp. 770–780, 1988.

[10] H. Dedieu, M. P. Kennedy, and M. Hasler, "Chaos shift keying: Modulation and demodulation of a chaotic carrier using self-synchronising Chua's circuits," *IEEE Trans. Circuits Syst.*, this issue.

[11] L. Fabiny and K. Wiesenfeld, "Clustering behavior of oscillator arrays," *Phys. Rev. A*, vol. 43, pp. 2640–648, 1991.

[12] W. J. Freeman, "Srange attractors that govern mammalian brain dynamics shown by trajectories of electroencephalographic (EEG) potential," *IEEE Trans. Circuits Syst.*, vol. 35, pp. 781–783, 1988.

[13] K. S. Halle, C. -W. Wu, M. Itoh, and L. O. Chua, "Spread spectrum communication through modulation of chaos," *Int. J. Bifurcation Chaos*, vol. 3, no. 2, pp. 469–477, 1993.

[14] S. Hayes, C. Grebogi, and E. Ott, "Communicating with chaos," *Phys. Rev. Lett.*, vol. 70, pp. 3031–3034, 1993.

[15] K. Kaneko, "Clustering, coding, switching, hierarchical ordering, and control in a network of chaotic elements, *Phys.* vol. D41, pp. 137–172, 1990.

[16] K. Kaneko, "Pattern dynamics in spatio-temporal chaos," *Phys.* vol. D34, pp. 1–41, 1989.

[17] M. P. Kennedy, "Robust op-amp realization of Chua's circuit," *Frequenz*, vol. 46, pp. 66–80, 1992.

[18] M. P. Kennedy and H. Dedieu, "Experimental demonstration of binary chaos—Shift-keying using self-synchronising Chua's circuits," preprint, 1992.

[19] L. Kočarev, K. S. Halle, K. Eckert, L. O. Chua, and U. Parlitz, "Experimental demonstration of secure communications via chaotic synchronisation," *Int. J. Bifurcation Chaos*, vol. 2, pp. 709–713, 1992.

[20] L. Kočarev, A. Shang, and L. O. Chua, "Transitions in dynamical regime by driving: A method of control and synchronisation of chaos," *Int. J. Bifurcation Chaos*, vol. 3, no. 2 pp. 479–483.

[21] F. Mossayebi, H. K. Qammar, and T. T. Hartley, "Adaptive estimation and synchronisation of chaotic systems," *Phys. Lett. A*, vol. 161, pp. 255–262, 1991.

[22] G. J. Mpitsos and R. M. Burton, "Convergence and divergence in neural networks: Processing of chaos and biological analogy," *Neural Networks*, vol. 5, pp. 605–625, 1992.

[23] V. I. Nekorkin and L. O. Chua, "Spatial disorder and wavefronts in a chain of coupled Chua's circuits," preprint, 1993.

[24] J. S. Nicolis, "Chaotic dynamics in biological information processing: A heuristic outline," in *Chaos in Biological Systems* (H. Degn, A. V. Holden and L. F. Olsen, Eds.). New York: Plenum, 1987, pp. 221-232.

[25] A. V. Oppenheim, G. W. Wornell, S. H. Isabelle, and K. M. Cuomo, "Signal processing in the context of chaotic signals," *Proc. IEEE ICASSP* (San Francisco, CA), 1992, pp. IV-117–IV-120, vol. 4.

[26] U. Parlitz, L. O. Chua, L. Kočarev, K. S. Halle, and A. Shang, "Transmission of digital signals by chaotic synchronisation," *Int. J. Bifurcation Chaos*, vol. 2, pp. 973–977, 1992.

[27] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," *Phys. Rev. Lett.*, vol. 64, pp. 821–824, 1991.

[28] ——, "Driving systems with chaotic signals," *Phys. Rev. A*, vol. 44, pp. 2374–2383, 1991.

[29] ——, "Synchronized Chaotic Signals and Systems," Proc. IEEE ICASSP, San Francisco, San Francisco, CA, vol. 4, pp. IV-137–IV-140, 1992.

[30] V. Perez-Munuzuri, V. Perez-Villar, and L. O. Chua, "Propagation failure in linear arrays of Chua's circuits," *Int. J. Bifurcation Chaos*, vol. 2, no. 2, pp. 403–406, 1992.

[31] ——, "Autowaves for image processing an a two-dimensional CNN array of Chua's circuits: Flat and wrinkled labyrinths," Memo. no. UCB/ERL M92/75, June 1, 1992.

[32] R. Pool, "Is it healthy to be chaotic?," *Sci.*, vol. 243, p. 604, 1989.

[33] N. F. Rul'kov, A. R. Volkovskii, A. Rodriguez-Lozano, E. del Rio, and M. G. Velarde, "Mutual synchronization of chaotic self-oscillators with dissipatiive coupling," *Int. J. Bifurcation Chaos*, vol. 2, pp. 669–676, 1992.

[34] M. de Sousa Vieira *et al.*, "Numerical and experimental studies of self-synchronization and synchronizsed chaos," *Int. J. Bifurcation Chaos*, vol. 2, pp. 645–657, 1992.

[35] G. S. Vernam, "Cipher printing telegraph systems for secret wire and telegraphic communications," *J. Amer. Inst. Elec. Eng.*, vol. 55, pp. 109–115, 1926.

[36] *Proc. IEEE*, Special Issue on Cryptography, May 1988.

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