DIGITAL SIMULATION OF NONLINEAR CIRCUITS BY WAVE DIGITAL FILTER PRINCIPLES

Klaus Meerkötter and Reinhard Scholz

Department of Electrical Engineering
University of Paderborn
4790 Paderborn, West Germany

ABSTRACT

The known principles of wave digital filters are used to simulate certain nonlinear circuits by digital means. As nonlinear elements, in particular, resistances having a continuous piecewise-linear voltage-current characteristic are considered. It turns out that a representation of a nonlinear resistance in terms of the wave variables may exist even if the voltage-current characteristic is neither voltage-controlled nor current-controlled. As an example, Chua's circuit is considered. It is shown that in the wave digital model of this circuit essentially the same double scroll attractor can be observed as in the corresponding analog realization.

INTRODUCTION

Every wave digital filter can be considered to be a digital model of its reference filter, i.e. the continuous-time filter from which the wave digital filter is derived [1]. In many respects, both the digital filter and its reference filter behave similarly. If, for example, a reference filter is stable, passive and/or lossless, the corresponding wave digital filter will be stable, (pseudo-)passive and/or (pseudo-)lossless, respectively. In principle, almost any type of continuous-time filter can be used as a reference filter. However, of particular importance are classical reactance filters in ladder or lattice configuration. Then, the resulting wave digital filter possess many interesting properties, which make these filters so attractive for practical applications [1].

The analogy between a wave digital filter and its reference filter is based on the following principles:

1. As frequency variable the well-known bilinear transform of the z-variable is used, i.e. the quantity

   $$\psi = (z-1)/(z+1) = \tanh(pT/2), \ z = e^{pT}, \qquad (1)$$

   where p denotes the actual complex frequency and T, the operating period of the digital filter.

2. Instead of voltages and currents so-called wave quantities are adopted as signal parameters. If v denotes the voltage and i the current at a port, the waves traveling in the forward and the backward directions are defined by

   $$a = v + Ri \quad \text{and} \quad b = v - Ri, \qquad (2)$$

   respectively, where R is a positive constant, the so-called port resistance.

Hereafter it is shown that these principles can not only be applied to linear filter circuits but also to certain nonlinear circuits. In particular, it will be demonstrated that chaotic phenomena, which may occur in a certain nonlinear continuous-time circuit [2]–[4], can be observed in the corresponding wave digital model as well.

LINEAR BUILDING BLOCKS

All linear elements used in classical passive filter circuits (resistances, capacitances, inductances, gyrators, circulators, ideal transformers, unit elements, and quasi-reciprocal lines) and resistive sources can be digitally simulated by obeying the principles mentioned in the Introduction [1]. As examples, let us consider only a resistance, a capacitance, and an inductance.

From the voltage-current characteristic of a resistance R_1, i.e. from $v = R_1 i$, we obtain

$$b = \frac{R_1 - R}{R_1 + R} a \, , \qquad (3)$$

where a and b are the incident and the reflected waves, respectively, which are defined by (2). If, as in most cases, the port resistance is chosen equal to the element value R_1, (3) reduces to

$$b = 0. \qquad (4)$$

The reactive elements such as the capacitance and the inductance are defined under steady-state conditions at an arbitrary complex frequency. Denoting the complex amplitudes of the voltage and the current by V and I, respectively, and using the equivalent frequency defined by (1), we may describe the capacitance and the inductance by

$$V = RI/\psi \quad \text{and} \quad V = \psi RI, \qquad (5)$$

respectively. If, in both cases, the port resistance is chosen equal to the element value, we obtain from (5)

$$B = z^{-1}A \quad \text{and} \quad B = -z^{-1}A, \qquad (6)$$

respectively, where A and B are the complex amplitudes of the incident and the reflected waves. The corresponding expressions for the instantaneous wave quantities are therefore given by

$$b(t) = a(t-T) \quad \text{and} \quad b(t) = -a(t-T). \qquad (7a,b)$$

Hence a capacitance is simulated by a simple delay, and an inductance, by a delay combined with a sign inverter.

In terms of voltage and current, equation (7a) can equivalently be expressed as

$$v(t) = v(t-T) + R\left[i(t) + i(t-T)\right]. \qquad (8)$$

Setting $R = T/(2C)$, we can easily check that (8) is exactly the same expression that would be obtained from the usual equation of a capacitance, i.e. from

$$v(t) = v(t-T) + \frac{1}{C}\int_{t-T}^{t} i(\tau)\mathrm{d}\tau, \qquad (9)$$

by applying the trapezoidal rule to the integral. A similar conclusion can be drawn for the inductance.

The simulation of the interconnections present in the reference filter is achieved by the so-called adaptors. These are n-port building blocks composed of adders and multipliers. An n-port parallel adaptor serves to simulate a parallel connection of n ports, and an n-port series adaptor, a corresponding series connection. In Fig. 1 there is shown an example of a three-port parallel adaptor and a corresponding signal flow diagram. As the reflected wave b_3 is independent of the incident wave a_3, port 3 is called reflection-free.

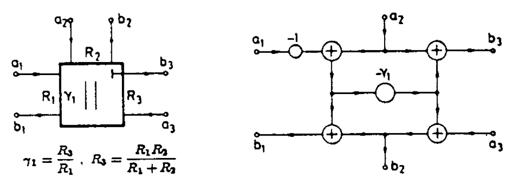

Fig. 1. Three-port parallel adaptor, with port 3 reflection-free, and signal-flow diagram.

NONLINEAR RESISTANCES

Consider a nonlinear resistance and assume first that the current i can be expressed as a (single-valued) function of the voltage v, i.e.

$$i = i(v),$$

where v can take on any real value. The incident and reflected waves at this resistance, which is said to be voltage-controlled, are given by

$$a = f(v) = v + Ri(v) \qquad (10a)$$

and

$$b = g(v) = v - Ri(v). \qquad (10b)$$

If f has an inverse, f^{-1}, defined on the real axis, we can obviously write

$$b = b(a) = g(f^{-1}(a)). \qquad (11)$$

It is easy to show that the unique invertibility of f, i.e. the existence of f^{-1}, is not only sufficient but also necessary for being able to express b as a function of a. In fact, if $f(v_1) = f(v_2)$ did not imply $v_1 = v_2$, we would have $g(v_1) \neq g(v_2)$.

Clearly, f^{-1} will exist if f is strictly monotonic. To guarantee the strict monotonicity of f, R has to be chosen such that

either $$\frac{1}{R} > -\inf_{v_1 \neq v_2} \frac{i(v_2) - i(v_1)}{v_2 - v_1} \qquad (12a)$$

or $$\frac{1}{R} < -\sup_{v_1 \neq v_2} \frac{i(v_2) - i(v_1)}{v_2 - v_1} \qquad (12b)$$

holds. If $i(v)$ is continuous and piecewise-differentiable with derivative $i'(v)$, (12a) and (12b) can be replaced by

$$1/R > -\inf_v i'(v) \qquad (13a)$$

and

$$1/R < -\sup_v i'(v), \qquad (13b)$$

respectively. As the port resistance R has to be positive, we will have to choose, in practically all cases, R according to (12a) or (13a) unless the suprema in (12b) and (13b) are negative.

As an example for a voltage-controlled resistance let us consider a continuous piecewise-linear function consisting of $n+1$ segments, which can be described by [5]

$$i(v) = i(0) + g_0 v + \sum_{\nu=1}^{n} g_\nu \left(|v - v_\nu| - |v_\nu|\right) \qquad (14)$$

where g_0, g_ν, and v_ν are real constants, with $v_{\nu-1} < v_\nu$ ($\nu = 2$ to n). Denoting the slope of $i(v)$ in the intervals $(-\infty, v_1)$, $(v_\nu, v_{\nu+1})$, and (v_n, ∞) by G_0, G_ν, and G_n, respectively, we may write

$$g_0 = (G_0 + G_n)/2$$

and

$$g_\nu = (G_\nu - G_{\nu-1})/2\ ,\ \nu = 1 \text{ to } n.$$

In order to make sure that a function $b = b(a)$, where a and b are defined by (10), can be derived from (14), the port resistance R has to meet one of the inequalities (13), i.e.

either $$1/R > -\inf_v i'(v) = -\min_\nu G_\nu$$

or $$1/R < -\sup_v i'(v) = -\max_\nu G_\nu.$$

If R is appropriately chosen, $b(a)$ turns out to be a continuous piecewise-linear function, which can be written in the form

$$b(a) = b(0) + c_0 a + \sum_{\nu=1}^{n} c_\nu \left(|a - a_\nu| - |a_\nu|\right),$$

where c_0, c_ν, a_ν, and $b(0)$ can be derived from the slopes G_ν and the constants appearing in (14) according to

$$c_0 = (\varrho_0 + \varrho_n)/2, \quad c_\nu = (\varrho_\nu - \varrho_{\nu-1})/2, \ \nu = 1 \text{ to } n$$

$$\varrho_\nu = (1 - RG_\nu)/(1 + RG_\nu), \ \nu = 0 \text{ to } n$$

$$a_\nu = v_\nu + Ri(v_\nu), \ \nu = 1 \text{ to } n$$

$$b(0) = -(1 + c_0)Ri(0) - \sum_{\nu=1}^{n} c_\nu \left(|Ri(0) - a_\nu| - |a_\nu|\right).$$

Next, we assume that the nonlinear resistance is current-controlled, i.e. that the voltage v is a (single-valued) function of the current i:

$$v = v(i).$$

The waves a and b are then given by

$$a = f_1(i) = v(i) + Ri$$

and

$$b = g_1(i) = v(i) - Ri.$$

To express b as a function of a, we have now to make sure that f_1 is invertible, in which case we may write

$$b = b(a) = g_1\left(f_1^{-1}(a)\right).$$

The strict monotonicity of f and thus its invertibility will be guaranteed if R meets

either $$R > -\inf_{i_1 \neq i_2} \frac{v(i_2) - v(i_1)}{i_2 - i_1}$$

or $$R < -\sup_{i_1 \neq i_2} \frac{v(i_2) - v(i_1)}{i_2 - i_1}.$$

In some cases, the nonlinear resistance will neither be voltage-controlled nor current-controlled. Nevertheless, a description in terms of the wave quantities may still be possible if an appropriate parametric representation of the voltage-current characteristic exists. As a simple example of a resistance having such a characteristic, we consider an ideal rectifier, which is usually defined by

$$(v \leq 0 \wedge i = 0) \vee (v = 0 \wedge i \geq 0).$$

For convenience, we will use the equivalent parametric representation

$$v = (\xi - |\xi|) \quad \text{and} \quad i = (\xi - |\xi|)/R, \qquad (15)$$

where ξ is a real parameter $(-\infty < \xi < \infty)$ and where R is an arbitrary positive constant, which will be chosen equal to the port resistance. From (2) and (15) we conclude that a corresponding parametric representation for the wave variables is thus given by

$$a = 2\xi \quad \text{and} \quad b = -2|\xi|.$$

We can therefore describe, for arbitrary port resistances, the ideal rectifier by the simple function

$$b = -|a|.$$

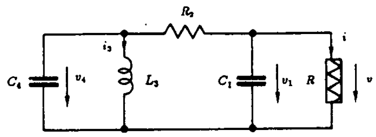

Fig. 2. Nonlinear circuit according to Ref. 2.

DIGITAL MODEL OF CHUA'S CIRCUIT

We consider the arrangement shown in Fig. 2, which has been suggested by Chua and studied by various authors (see e.g. [2]–[4]). It contains four linear one-port elements and one nonlinear resistance denoted by R and having the characteristic (Fig. 3)

$$i = G_1 v + \frac{1}{2}(G_2 - G_1)\left(|v + v_0| - |v - v_0|\right), \qquad (16)$$

with $G_1 = -500\ \mu S$, $G_2 = -800\ \mu S$, $v_0 = 1$ V. The element values of the linear elements are given by

$$C_1 = 5.5\text{nF}, \ R_2 = 1.428\text{k}\Omega, \ L_3 = 7.07\text{mH}, \ C_4 = 49.5\text{nF}.$$

The wave flow diagram derived from the circuit of Fig. 2 is depicted in Fig. 4. In order to be able to determine the coefficients of the series adaptor and the two parallel adaptors, we have first to fix the port resistances of those ports that are terminated by the reactive elements. Clearly, this must be done by taking into account the operating period T, which has been chosen equal to 10 μs. In general, the choice of T should be made in such a way that, in the frequency range of interest, the impedance of a wave digital reactive element (i.e. ψR or R/ψ) approximates the impedance of the corresponding analog lumped element (i.e. pL or $1/pC$). (Note that the resonant frequency of the parallel circuit consisting of L_3 and C_4 is given by 8.5 kHz.)

The port resistances are then to be fixed according to

$$R_1 = T/(2C_1), \quad R_3 = 2L/T, \quad \text{and } R_4 = T/(2C_4).$$

From these resistances and from R_2, as given above, the following numerical values for the adaptor coefficients have been calculated:

$$\gamma_1 = 0.066673, \quad \gamma_2 = 0.061931, \quad \gamma_3 = 0.373901.$$

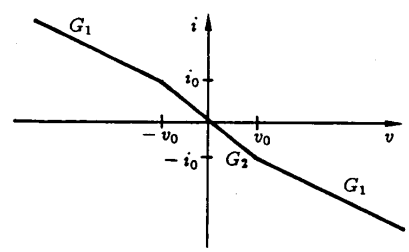

Fig. 3. Characteristic of the nonlinear resistance defined by (16).

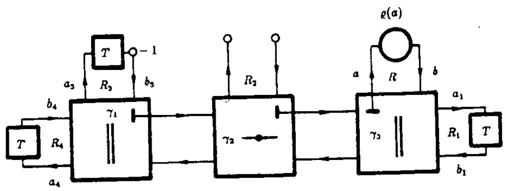

Fig. 4. Wave digital model of the circuit of Fig. 2.

Due to the fact that the port terminated by the nonlinear element has to be reflection-free, the corresponding port resistance cannot be chosen arbitrarily but is determined by the other port resistances and is given by

$$R = 569.2\ \Omega.$$

Using this resistance, we obtain from (16) for the characteristic of the nonlinear element the expression

$$b = \varrho(a) = \varrho_1 a + \frac{1}{2}(\varrho_2 - \varrho_1)(|a + a_0| - |a - a_0|), \quad (17)$$

with

$$\varrho_1 = (1 - G_1 R)/(1 + G_1 R) = 1.7956,$$
$$\varrho_2 = (1 - G_2 R)/(1 + G_2 R) = 2.6722,$$
$$a_0 = v_0(1 + G_2 R) = 0.5447\ \text{V}.$$

This characteristic is plotted in Fig. 5.

Finally, in Fig. 6 there are shown some projections of the trajectory, which has been observed in the digital model of Fig. 4. These projections are very similar to those published in Ref. 3. It is interesting to note that an increase of the operating period T by a factor of two has not essentially changed the trajectory.

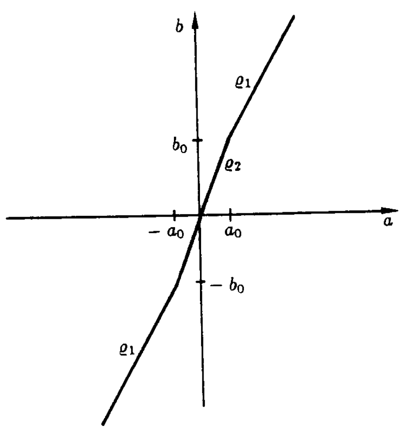

Fig. 5. Plot of the characteristic defined by (17).

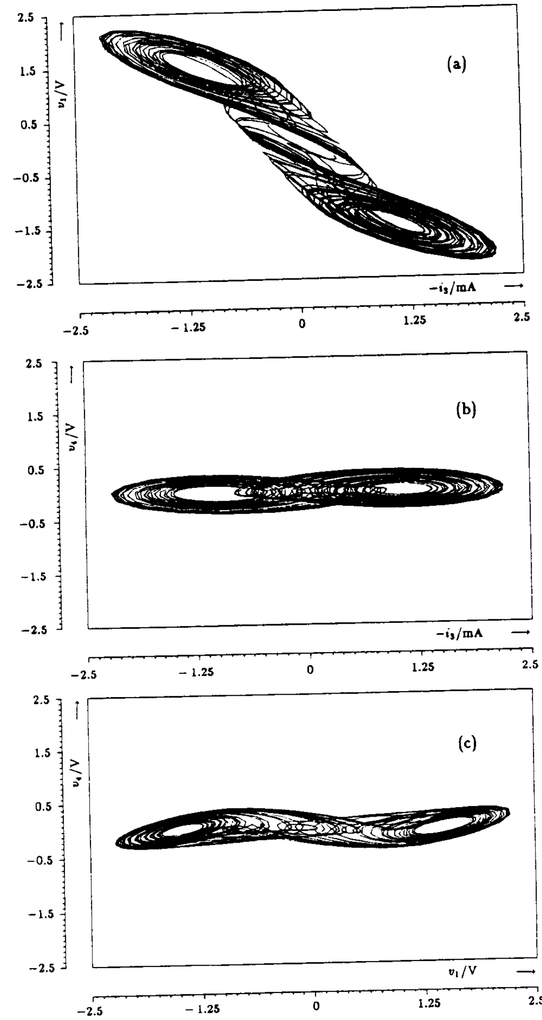

Fig. 6. The observed chaotic attractor.

(a) Projection onto the $(-i_3, v_1)$-plane.

(b) Projection onto the $(-i_3, v_4)$-plane.

(c) Projection onto the (v_1, v_4)-plane.

REFERENCES

[1] A. Fettweis, "Wave digital filters: theory and practice," Proc. IEEE, vol. 74, pp. 270-327, 1986.

[2] T. Matsumoto, "A chaotic attractor from Chua's circuit," IEEE Trans. CAS, vol. CAS-31, pp. 1055-1058, Dec. 1984.

[3] T. Matsumoto, L. Chua, M. Komuro, "The double scroll," IEEE Trans. CAS, vol. CAS-32, pp. 798-818, Aug. 1985.

[4] C. Silva, L. Chua, M. Komuro, "The overdamped double scroll family," IEEE Int. J. CTA, vol. CTA-16, pp. 233-302, 1988.

[5] L. Chua, S. Kang, "Section-wise piecewise-linear functions: Canonical representation, properties, and applications," Proc. IEEE, (Special Issue on Multidimensional Systems), vol. 65, pp. 915-929, June 1977.