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CONTROLLING CHAOTIC DYNAMICS USING BACKSTEPPING DESIGN WITH APPLICATION TO THE LORENZ SYSTEM AND CHUA'S CIRCUIT

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In this Letter backstepping design is proposed for controlling chaotic systems. The tool consists in a recursive procedure that combines the choice of a Lyapunov function with the design of feedback control. The advantages of the method are the following: (i) it represents a systematic procedure for controlling chaotic or hyperchaotic dynamics; (ii) it can be applied to several circuits and systems reported in literature; (iii) stabilization of chaotic motion to a steady state as well as tracking of any desired trajectory can be achieved. In order to illustrate the general applicability of backstepping design, the tool is utilized for controlling the chaotic dynamics of the Lorenz system and Chua's circuit. Finally, numerical simulations are carried out to show the effectiveness of the technique.

1. Introduction

Controlling chaotic circuits and systems has received great interest in recent years [Ott *et al.*, 1990; Hunt, 1991; Gills *et al.*, 1992; Chen & Dong, 1993; Fuh & Tung, 1995; Yang & Chua, 1997; Srivastava & Srivastava, 1998]. As is well known, Ott, Grebogi and Yorke (OGY) were the first to introduce a technique that can stabilize a periodic orbit embedded in a chaotic attractor [Ott *et al.*, 1990]. However, since steady-state solutions represent the most practical operation mode in many chaotic systems such as electronic circuits [Huang *et al.*, 1996; Yang & Chua, 1997] or laser systems [Gills *et al.*, 1992], it is important to develop control techniques able to drive a strange attractor not only to a periodic orbit but also to a steady state. Moreover, another desirable feature is to develop techniques that are not closely related to the particular chaotic system to be controlled. Some attempts to solve these problems have been made using occasional proportional feedback (OPF) techniques [Hunt, 1991; Inaba & Nitanai, 1998; Tsubone & Saito, 1998]. However, a theoretical analysis of the technique developed in [Hunt, 1991] is hard to be carried out, whereas the OPF techniques proposed in [Inaba & Nitanai, 1998] and [Tsubone & Saito, 1998] can be applied only to the class of piecewise-linear chaotic systems.

In this Letter backstepping design is proposed for controlling chaotic systems. The suggested tool enables *stabilization* of chaotic motion to a steady state as well as *tracking* of any desired trajectory to be achieved in a *systematic* way.

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Furthermore, it can be applied for controlling the chaotic (hyperchaotic) dynamics of several well-known circuits and systems [Rössler, 1976; 1979; Lorenz, 1963; Huang *et al.*, 1996; Tamasevicius *et al.*, 1996a; 1996b; Namajunas & Tamasevicius, 1996; Tamasevicius, 1997; Saito, 1990]. The Letter is organized as follows. In Sec. 2 the class of strict-feedback systems is presented and the basic notions of backstepping design are illustrated [Krstic *et al.*, 1995]. In order to show the general applicability of the technique, in Sec. 3 the tool is utilized for achieving stabilization and tracking in the Lorenz system, whereas in Sec. 4 it is applied for stabilizing Chua's circuit. Numerical simulations are carried out to confirm the validity of the proposed theoretical approach. Moreover, a comparison with the differential geometric method [Isidori, 1995] is made. Finally, the robustness with respect to noise and the behavior in the presence of disturbance are investigated.

2. Backstepping Design

Backstepping design is a systematic Lyapunov-based control technique, which can be applied to strict-feedback systems, pure-feedback systems and block-strict-feedback systems [Krstic *et al.*, 1995]. In this Letter the attention is focused on strict-feedback systems, since this class includes several examples of chaotic circuits and systems. Namely, let

$$\dot{x} = f(x) + g(x)\xi_1 \tag{1}$$

be an input affine nonlinear system, where $x \in \Re^n$ is the state, $\xi_1 \in \Re$ is the scalar control input whereas f and g are nonlinear functions, with $f(0) = 0$. Let the system (1) be augmented by the following equations:

$$\begin{aligned}
\dot{x} &= f(x) + g(x)\xi_1 \\
\dot{\xi}_1 &= f_1(x,\, \xi_1) + g_1(x,\, \xi_1)\xi_2 \\
&\vdots \\
\dot{\xi}_{k-1} &= f_{k-1}(x,\, \xi_1, \ldots,\, \xi_{k-1}) \\
&\quad + g_{k-1}(x,\, \xi_1, \ldots,\, \xi_{k-1})\xi_k \\
\dot{\xi}_k &= f_k(x,\, \xi_1, \ldots,\, \xi_k) + g_k(x,\, \xi_1, \ldots,\, \xi_k)u
\end{aligned} \tag{2}$$

where $\xi_1, \xi_2, \ldots, \xi_k$ and u are scalars [Krstic *et al.*, 1995]. Systems in the form (2) are said to be *strict-feedback systems* because the nonlinearities f_i and g_i in the $\dot{\xi}_i$-equation ($i = 1, \ldots, k$) depend only on x, $\xi_1, \xi_2, \ldots, \xi_i$, that is, on state variables that are "fed back". Notice that several chaotic systems belong to the class described by (2). These systems are Rössler's chaotic system [Rössler, 1976], the Lorenz system [Lorenz, 1963], Chua's circuit [Yang & Chua, 1997; Huang *et al.*, 1996], the chaotic circuits in [Tamasevicius *et al.*, 1996a; Namajunas & Tamasevicius, 1996; Tamasevicius, 1997]. Regarding hyperchaotic systems (i.e. systems with more than one positive Lyapunov exponent), the class of strict-feedback systems includes Rössler's hyperchaotic system [Rössler, 1979] and the hyperchaotic circuits in [Saito, 1990; Tamasevicius *et al.*, 1996b].

Backstepping design consists in a recursive procedure that interlaces the choice of a Lyapunov function with the design of feedback control [Krstic *et al.*, 1995; Khalil, 1996]. The key idea is to utilize the Lyapunov's method by breaking the design problem for the full system (2) into a sequence of design problems for lower-order (even scalar) systems. The technique starts by considering the variable ξ_1 as a "virtual control input" to stabilize the first equation. And when ξ_1 has been designed, it continues to consider the variable ξ_2 as the virtual control for the second equation, and so on. Therefore the design of the actual input u, which usually depends on x and $\xi_1, \xi_2, \ldots, \xi_k$, is systematically achieved in n steps [Krstic *et al.*, 1995; Khalil, 1996]. It is worth noting that backstepping design proves to be particularly suitable for controlling chaos. Namely, it can solve stabilization and tracking problems under conditions less restrictive that those encountered in other techniques, since it exploits the flexibility assured by lower-order and scalar systems. Moreover, the technique gives the flexibility to build a control law by avoiding cancellations of useful nonlinearities. In this way the goals of stabilization and tracking are achieved with a reduced control effort.

3. Controlling Chaos in the Lorenz System

In order to show how backstepping design works, in this section the tool is applied for controlling the chaotic dynamics of the Lorenz system. In particular, both stabilization and tracking are achieved and a comparison with the differential geometric method [Fuh & Tung, 1995] is carried out.

3.1. *The Lorenz equations*

The system considered herein is described by the following set of dynamic equations [Singer *et al.*, 1991; Fuh & Tung, 1995]:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -px + py \\ -xz - y \\ xy - z - R \end{pmatrix} \tag{3}$$

where R is the Rayleigh number and $p = 10$ is the Prandtl number. It is assumed that only the parameter R can be modified. Thus, let the Rayleigh number be $R = R_0 + u$, where R_0 is the operation value and u is the control variable. For $R_0 = 28$ the uncontrolled system (i.e. $u = 0$) is chaotic. A projection of the Lorenz attractor on (x, y, z) is reported in Fig. 1. In this case, there are three unstable equilibrium points: $(C_0, C_0, -1)$, $(0, 0, -R_0)$ and $(-C_0, -C_0, -1)$, where $C_0 = \sqrt{R_0 - 1}$. It is worth noting that when the set point is the state $(C_0, C_0, -1)$ the OGY method is not applicable [Fuh & Tung, 1995]. By translating the origin of system (3) in the set point $(C_0, C_0, -1)$, the system equations become:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -px_1 + px_2 \\ x_1 - x_2 - (C_0 + x_1)x_3 \\ C_0(x_1 + x_2) - x_3 + x_1x_2 - u \end{pmatrix} \tag{4}$$

3.2. *Stabilization*

The objective is to find a control law u for *stabilizing* the state of system (4) at the origin. Starting from the first equation, a *stabilizing function* $\alpha_1(x_1)$ has to be designed for the *virtual control* x_2 in order

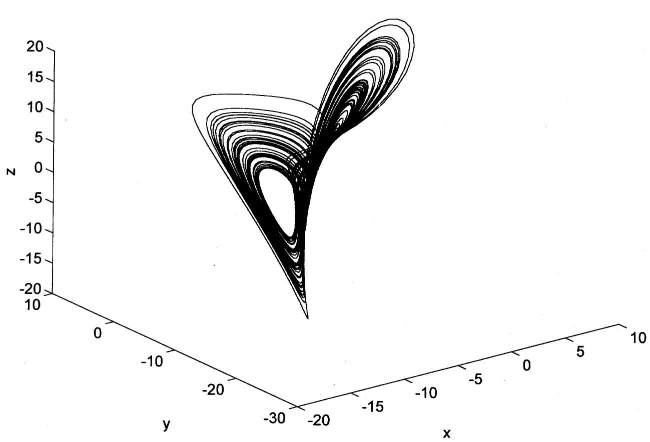


Fig. 1. Projection on (x, y, z) of the attractor generated by the Lorenz system.

to make the derivative of $V_1(x_1) = x_1^2/2$, that is:

$$\dot{V}_1 = -px_1^2 + px_1x_2\,,$$

negative definite when $x_2 = \alpha_1(x_1)$. By choosing $\alpha_1(x_1) = 0$ and by defining the *error variable* z_2 as:

$$z_2 = x_2 - \alpha_1(x_1) \tag{5}$$

the following (x_1, z_2)-subsystem is obtained:

$$\dot{x}_1 = -px_1 + pz_2$$

$$\dot{z}_2 = x_1 - z_2 - (C_0 + x_1)x_3$$

for which a candidate Lyapunov function is $V_2(x_1, z_2) = V_1(x_1) + \frac{1}{2}z_2^2$. Since its time derivative:

$$\dot{V}_2 = -px_1^2 + z_2[(p+1)x_1 - z_2 - (C_0 + x_1)x_3]$$

becomes negative definite by choosing the virtual control x_3 as:

$$x_3 = \alpha_2(x_1, z_2) = \frac{(p+1)x_1}{C_0 + x_1}$$

the deviation of x_3 from the stabilizing function α_2:

$$z_3 = x_3 - \frac{(p+1)x_1}{C_0 + x_1} \tag{6}$$

gives the following system in the (x_1, z_2, z_3) coordinates:

$$\dot{x}_1 = -px_1 + pz_2$$

$$\dot{z}_2 = x_1 - z_2 - (C_0 + x_1)(z_3 + \alpha_2)$$

$$\dot{z}_3 = [C_0(x_1 + z_2) - z_3 - \alpha_2 + x_1z_2 - u] - \left[\frac{(p+1)C_0}{(C_0 + x_1)^2}(-px_1 + pz_2)\right].$$

By iterating the previous steps, the derivative of $V_3(x_1, z_2, z_3) = V_2 + \frac{1}{2}z_3^2$, that is,

$$\dot{V}_3 = -px_1^2 - z_2^2 + z_3\left[C_0x_1 - z_3 - \frac{(p+1)x_1}{C_0 + x_1} - u - \frac{(p+1)C_0(-px_1 + px_2)}{(C_0 + x_1)^2}\right],$$

becomes negative definite by choosing the input:

$$u = C_0x_1 - \frac{(p+1)x_1}{C_0 + x_1} - \frac{(p+1)C_0(-px_1 + px_2)}{(C_0 + x_1)^2}\,, \tag{7}$$

which proves that the origin has been stabilized in the (x_1, z_2, z_3) coordinates. In view of (5) and (6), the origin in the (x_1, x_2, x_3) coordinates has the same properties. As a consequence, (7) represents the control law for stabilizing system (3) in $(C_0, C_0, -1)$.

3.3. *Tracking*

The goal is to find a control law u so that a scalar output $y(t)$ tracks any desired trajectory $r(t)$, including stable or unstable limit cycles as well as chaotic trajectories.

Let $y(t) = x_2$ be the output and let z_2 be the deviation of x_2 from the target, i.e. $z_2 = x_2 - r(t)$. Given $V_2 = z_2^2/2$, its time derivative:

$$\dot{V}_2 = z_2[x_1 - z_2 - r(t) - (C_0 + x_1)x_3 - \dot{r}(t)]$$

becomes negative by choosing the virtual control x_3 as:

$$x_3 = \alpha_2 = \frac{x_1 - r - \dot{r}}{C_0 + x_1}\,.$$

Again, given $V_3 = V_2 + z_3^2/2$, where $z_3 = x_3 - \alpha_2$ is the deviation of the virtual control from the stabilizing function, the time derivative:

$$\begin{aligned}\dot{V}_3 = -z_2^2 - z_3[&C_0 z_2 + z_2 x_1 - C_0(x_1 + x_2) \\ &+ z_3 + \alpha_2 - x_1 x_2 + u + \dot{\alpha}_2]\end{aligned}$$

is negative by choosing the input:

$$\begin{aligned}u = &\frac{(px_1 - px_2)(C_0 + r + \dot{r})}{(C_0 + x_1)^2} + \frac{2\dot{r} + \ddot{r} - x_1 + r}{C_0 + x_1} \\ &+ r(C_0 + x_1) + C_0 x_1 \end{aligned} \tag{8}$$

which assures that $y(t) = x_2(t)$ tracks the reference signal $r(t)$. Similar results can be obtained by choosing $x_1(t)$ or $x_3(t)$ as output.

Remark. Notice that, although Eqs. (7) and (8) could appear complex, they enable system (3) to be controlled using a single input. Different approaches can be exploited if we assume that the system can be controlled by two scalar inputs. For instance, if we had the system:

$$\begin{pmatrix}\dot{x} \\ \dot{y} \\ \dot{z}\end{pmatrix} = \begin{pmatrix}-px + py \\ -xz - y + R_1 \\ xy - z + R_2\end{pmatrix} \tag{9}$$

then (9) would be transformed to a linear system using the control laws $R_1 = xz + u_1$ and $R_2 = -xy + u_2$. As a consequence, linear techniques could be applied for controlling (9) via u_1 and u_2 [Kailath, 1980].

3.4. *Simulation results*

Numerical simulations are carried out to show the effectiveness of backstepping design. In particular, a comparison with the differential geometric method [Isidori, 1995; Fuh & Tung, 1995; Nijmeijer & van der Schaft, 1990] is made. Successively, the robustness with respect to noise and the behavior in the presence of disturbance are investigated.

Regarding stabilization, numerical simulations obtained using the backstepping design are reported in Fig. 2 for $p = 10$ and $C_0 = \sqrt{27}$. The control law (7) is switched on at $t = 20$. By considering the results reported in [Fuh & Tung, 1995], the control law using the differential geometric approach is given by:

$$u = -\frac{q(x)}{s(x)} + \frac{v(x)}{s(x)} \tag{10}$$

where:

$$\begin{aligned}q(x) = &(p^2 + p - px_3)(-px_1 + px_2) \\ &- (p^2 + p)(x_1 - x_2 - C_0 x_3 - x_1 x_3)x_2 \\ &- (pC_0 + px_1)(C_0 x_1 + C_0 x_2 - x_3 + x_1 x_2) \\ v(x) = &-1000x_1 - 215(-px_1 + px_2) \\ &- 17.5((p^2 + p)(x_1 - x_2) - pC_0 x_3 - px_1 x_3) \\ s(x) = &\, p(x_1 + C_0)\,.\end{aligned}$$

The results for the stabilization using the differential geometric method are shown in Fig. 3. With reference to tracking, the results for $r(t) = \sin(t)$ using the backstepping design are reported in Fig. 4. The control law using the differential geometric approach is given by [Fuh & Tung, 1995]:

$$u = -\frac{q(x)}{s(x)} + \frac{v(x) + r(t)}{s(x)} \tag{11}$$

and the corresponding results are shown in Fig. 5. It can be noted that backstepping design requires less control effort than differential geometric method. Moreover, the control laws (7) and (8) are simpler than the corresponding laws (10) and (11).

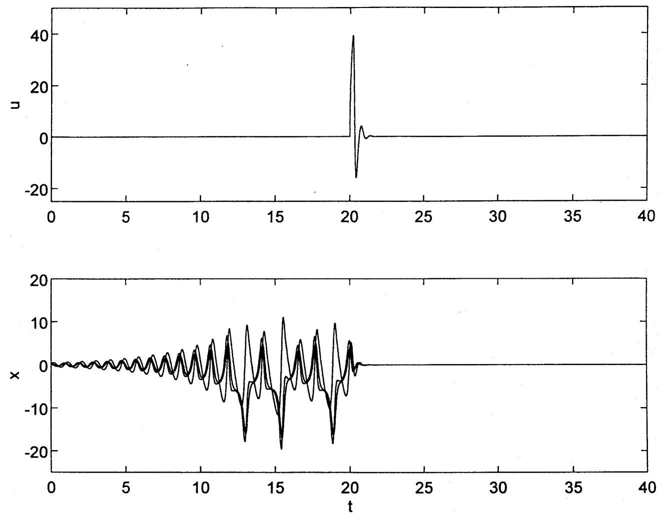


Fig. 2. Backstepping design applied to the Lorenz system: (a) time waveform of the control u switched on at $t = 20$; (b) stabilization of the state variables x_1, x_2, and x_3.

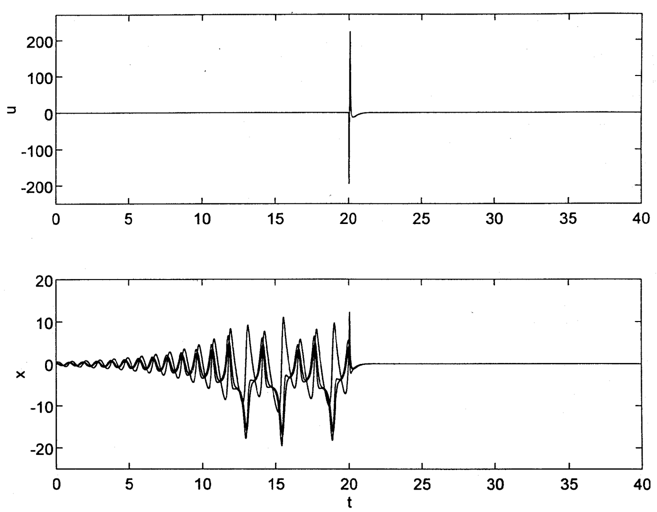


Fig. 3. Differential geometric method applied to the Lorenz system: (a) time waveform of the control u switched on at $t = 20$; (b) stabilization of the state variables x_1, x_2, and x_3.

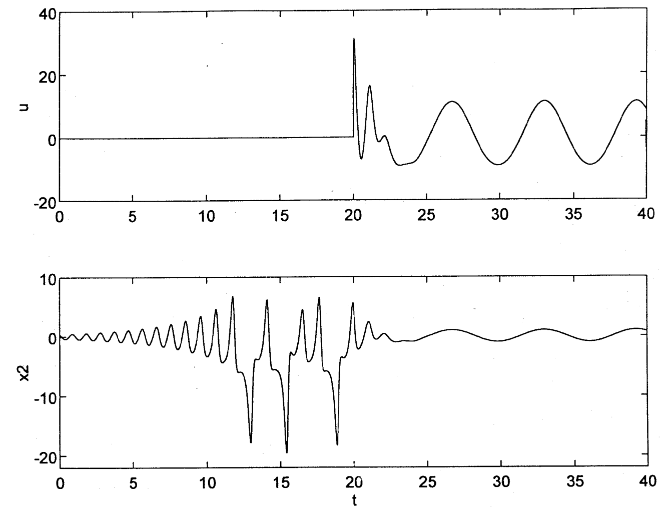


Fig. 4. Tracking of $r(t) = \sin(t)$ using the backstepping design: (a) time waveform of the control u switched on at $t = 20$; (b) time waveform of the output x_2 of the Lorenz system.

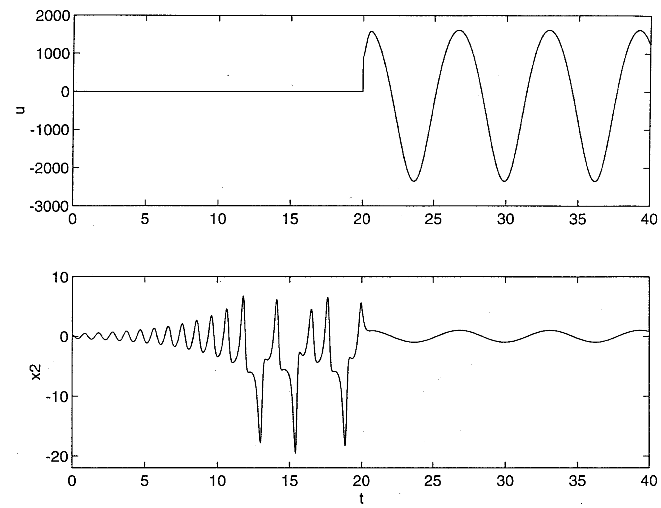


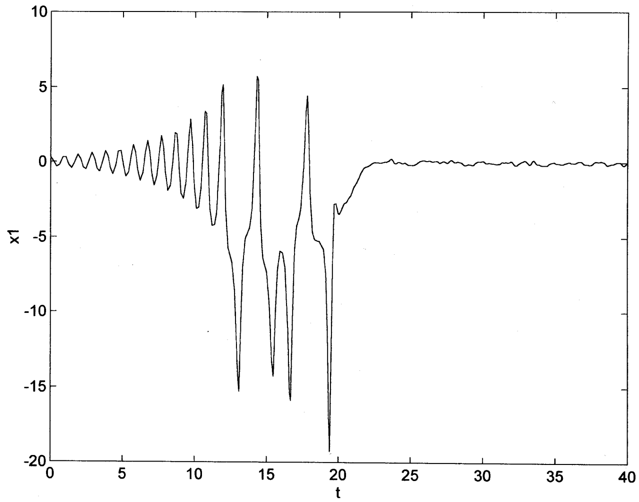
Fig. 5. Tracking of $r(t) = \sin(t)$ using the differential geometric method: (a) time waveform of the control u switched on at $t = 20$; (b) time waveform of the output x_2 of the Lorenz system.

This is because backstepping pursues the goals of stabilization and tracking rather than that of linearization.

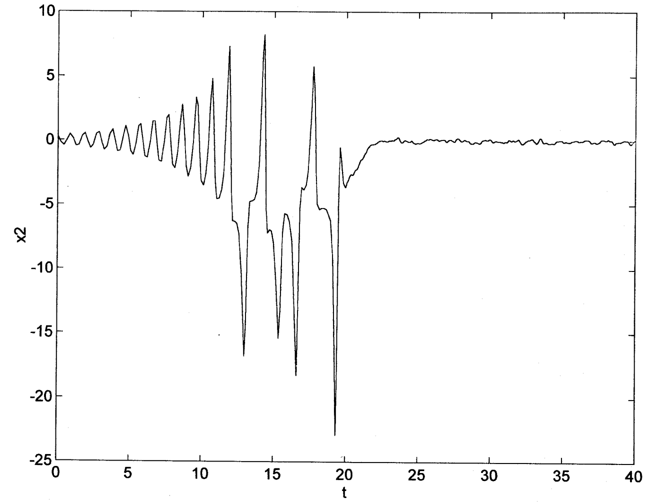
Now, the robustness with respect to noise is investigated. To this purpose, let x_1', x_2' and x_3' be the assumed measurements of the system states, that is:

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \tag{12}$$

where n_1, n_2 and n_3 are measurement noises. The measurements of the state variables (12) are used for the control law of both backstepping design and geometric approach. In particular, simulations are carried out by choosing for the noises a simple Gaussian distribution with zero mean and fixed variance (σ^2), that is, n_1, n_2, $n_3 \sim N(0, 0.2^2)$. The results for the stabilization using backstepping design and differential geometric method are reported in Figs. 6 and 7, respectively. Referring to the behavior in the presence of disturbance, let the Lorenz system be



(a)



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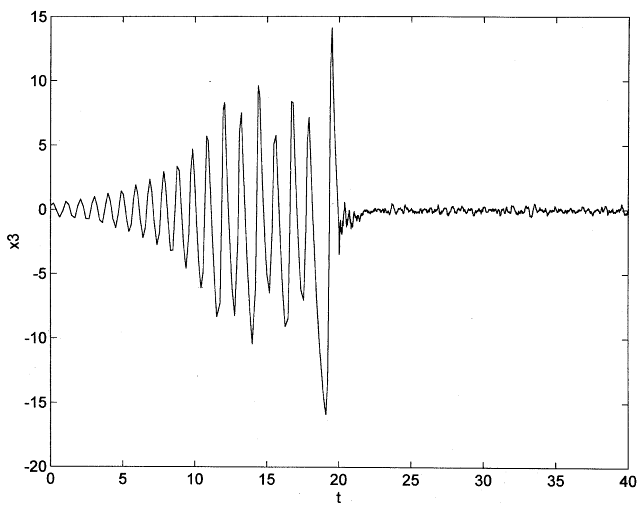


(c)

Fig. 6. Stabilization for the Lorenz system using the backstepping design. The control is obtained using noisy measurements (12) and is switched on at $t = 20$: (a) time waveform of x_1; (b) time waveform of x_2; (c) time waveform of x_3.

described as:

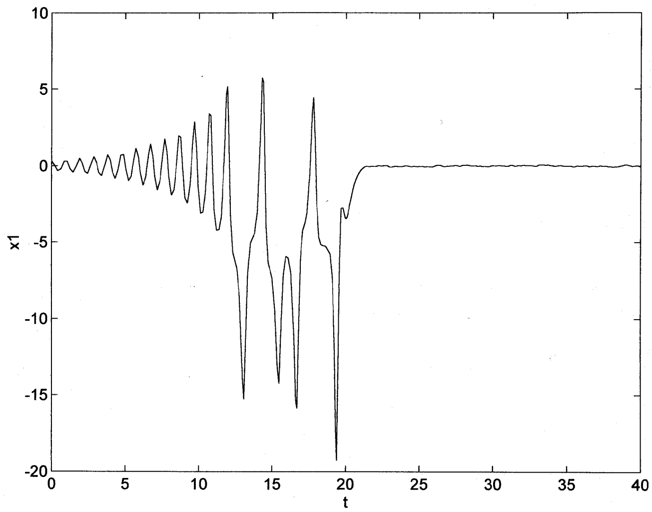
$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -px_1 + px_2 \\ x_1 - x_2 - (C_0 + x_1)x_3 \\ C_0(x_1 + x_2) - x_3 + x_1x_2 - u \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} d \tag{13}$$

where the disturbance d is a simple Gaussian distribution with zero mean and fixed variance, that is, $d \sim N(0, 02^2)$. Notice that the disturbance is added to the second and third equation, which contain the nonlinear terms. In this way, the disturbance can also model perturbation of the nonlinear terms. Regarding stabilization of system (13), the time waveforms of x_1, x_2 and x_3 using backstepping design and differential geometric method are reported in Figs. 8 and 9, respectively.

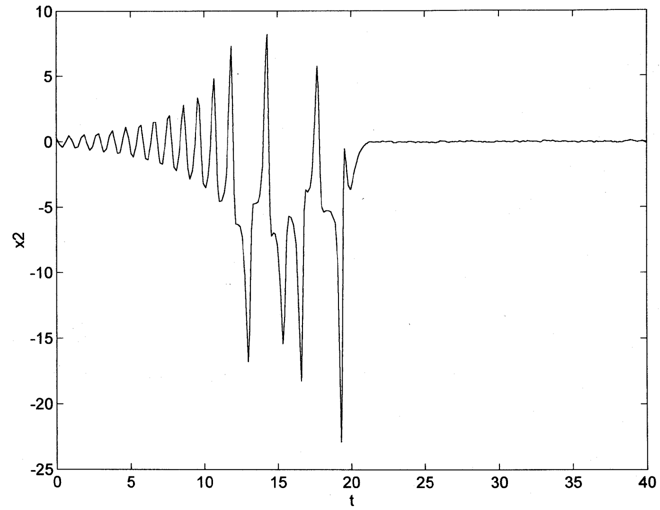
By analyzing Figs. 6–9, it can be observed that differential geometric approach is more robust with respect to noise and disturbance than backstepping design. However, notice that this result is obtained using a great control effort.

4. Controlling Chaos in Chua's Circuit

In order to show the general applicability of backstepping design, the attention is now focused on



(a)



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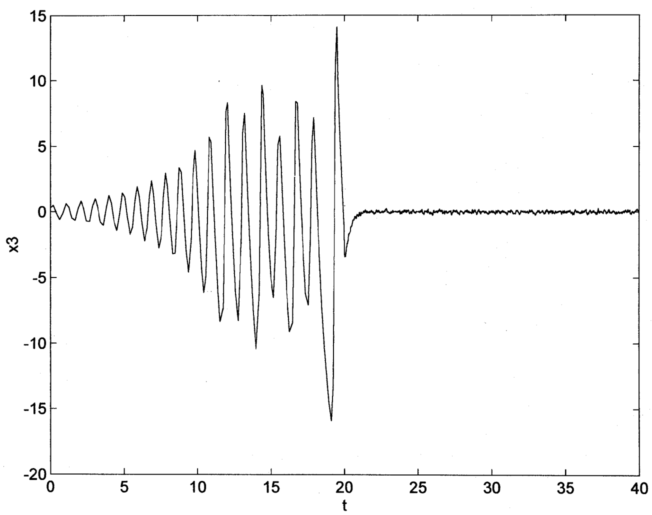


(c)

Fig. 7. Stabilization for the Lorenz system using the differential geometric method. The control is obtained using noisy measurements (12) and is switched on at $t = 20$: (a) time waveform of x_1; (b) time waveform of x_2; (c) time waveform of x_3.

Chua's circuit, which was the first physical dynamical system capable of generating chaotic phenomena in the laboratory [Madan, 1993]. The circuit considered herein contains a cubic nonlinearity and is described by the following set of differential equations [Huang *et al.*, 1996]:

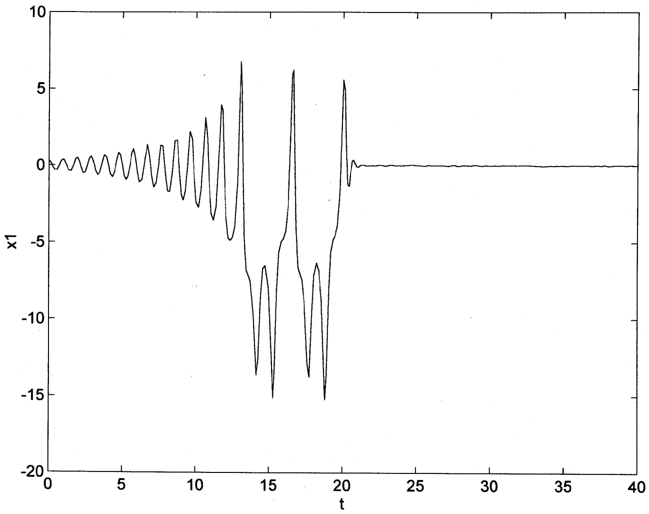
$$\begin{aligned} \dot{x} &= \alpha(y - x^3 - cx) \\ \dot{y} &= x - y + z \\ \dot{z} &= -\beta y \end{aligned} \tag{14}$$

where $\alpha = \alpha_0 u$, u is the scalar control variable whereas α_0, β and c are the circuit parameters. Notice that the parameter α can be easily modified by varying a single capacitance of Chua's circuit (see [Huang *et al.*, 1996]). By taking $\alpha_0 = 10$, $\beta = 16$ and $c = -0.143$ the uncontrolled circuit (that is, $u = 1$) exhibits the double-scroll attractor (Fig. 10). The objective is to find a control law u so that the chaotic dynamics of the circuit are stabilized at the origin. Starting from the third equation of system (14), a stabilizing function $\alpha_1(z)$ has to be designed for the virtual control y in order to make the derivative of $V_1(z) = z^2/2$, that is:

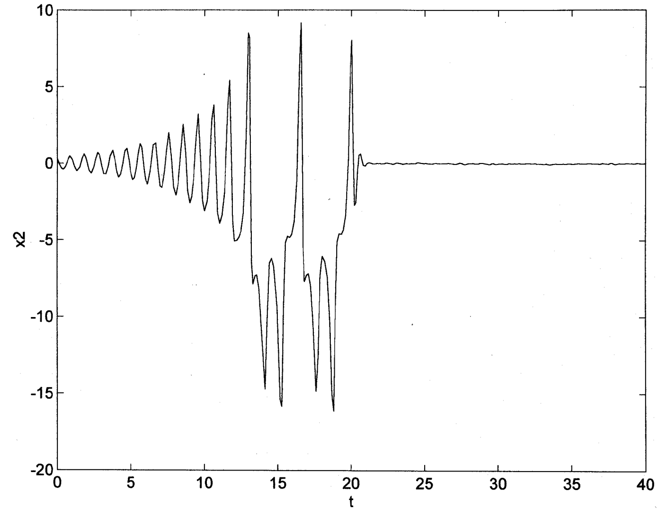
$$\dot{V}_1 = -\beta yz\,,$$

negative definite when $y = \alpha_1(z) = z$. By defining the error variable $\overline{y}$ as:

$$\overline{y} = y - \alpha_1(z) \tag{15}$$



(a)



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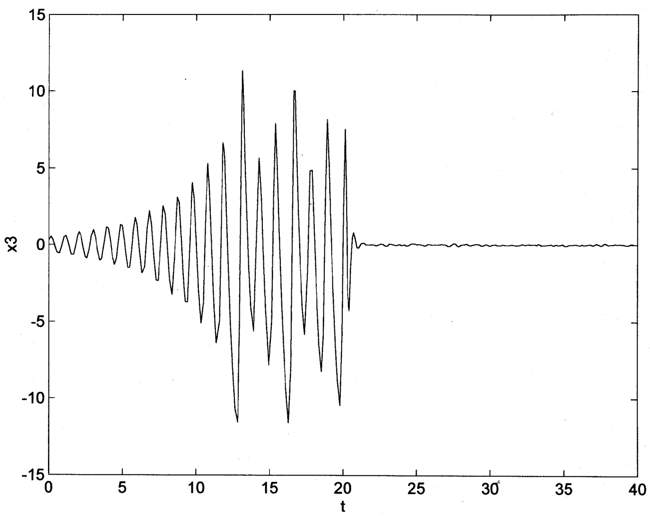


(c)

Fig. 8. Stabilization for system (13) using the backstepping design, with the control u switched on at $t = 20$: (a) time waveform of x_1; (b) time waveform of x_2; (c) time waveform of x_3.

the following $(\overline{y}, z)$-subsystem is obtained:

$$\dot{\overline{y}} = x - \overline{y} + \beta\,\overline{y} + \beta z$$
$$\dot{z} = -\beta(\overline{y} + z)$$

for which a candidate Lyapunov function is $V_2(\overline{y}, z) = V_1(z) + \frac{1}{2}\overline{y}^2$. Since its time derivative:

$$\dot{V}_2 = -\beta z^2 - \overline{y}^2 + \beta\,\overline{y}^2 + \overline{y}x$$

becomes negative definite by choosing the virtual control x as:

$$x = \alpha_2(\overline{y}, z) = -\beta\,\overline{y}$$

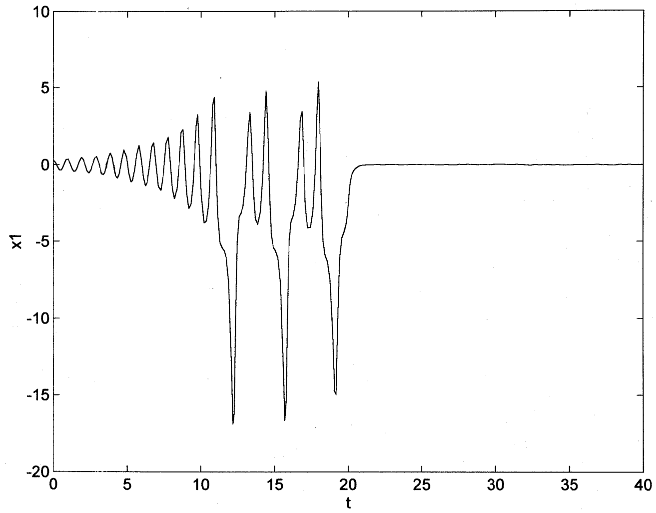
then the deviation of x from the stabilizing function α_2:

$$\overline{x} = x + \beta\,\overline{y} \tag{16}$$

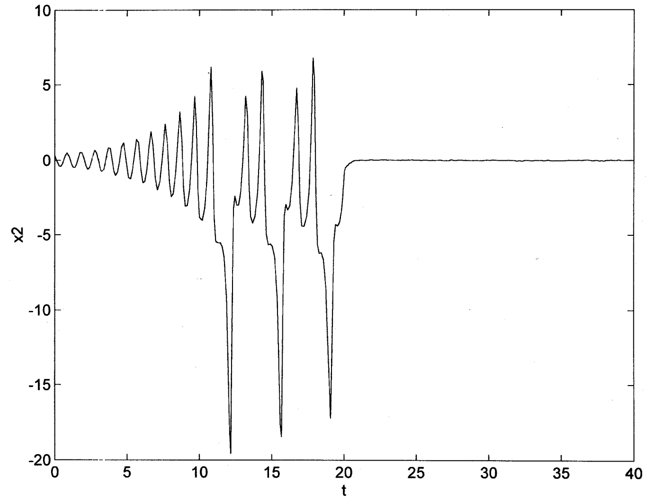
gives the following system in the $(\overline{x}, \overline{y}, z)$ coordinates:

$$\begin{aligned}\dot{\overline{x}} &= \alpha_0 u((\overline{y} + z) - (\overline{x} - \beta\,\overline{y})^3 - c(\overline{x} - \beta\,\overline{y})) \\ &\quad + \beta(\overline{x} - \overline{y} + \beta z) \\ \dot{\overline{y}} &= \overline{x} - \overline{y} + \beta z \\ \dot{z} &= -\beta(\overline{y} + z)\end{aligned}$$

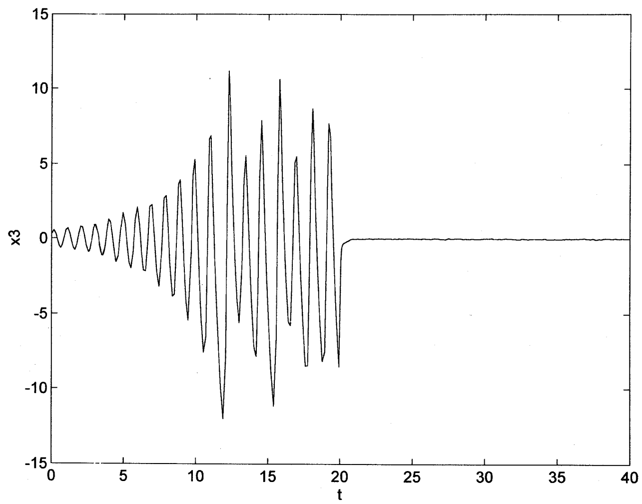
Finally, the derivative of $V_3(\overline{x}, \overline{y}, z) = V_2 + \frac{1}{2}\overline{x}^2$,



(a)



(b)



(c)

Fig. 9. Stabilization for system (13) using the differential geometric method, with the control u switched on at $t = 20$: (a) time waveform of x_1; (b) time waveform of x_2; (c) time waveform of x_3.

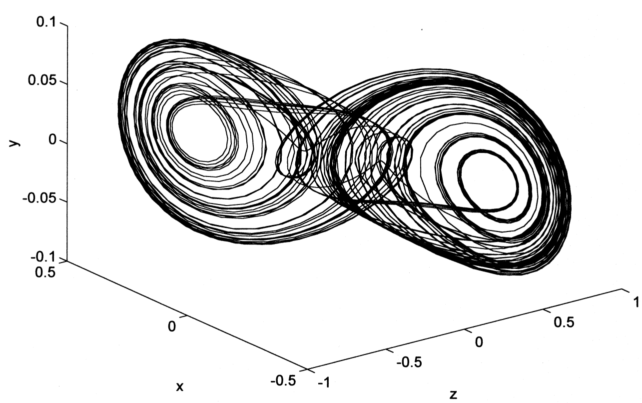


Fig. 10. Projection on (x, y, z) of the attractor generated by Chua's circuit.

that is,

$$\dot{V}_3 = -\beta z^2 - \overline{y}^2 + \overline{xy} + \overline{x}\alpha_0 u(y - x^3 - cx) + \overline{x}\beta(\overline{x} - \overline{y} + \beta z)\,,$$

becomes negative definite by choosing the control variable as

$$u = \frac{-\overline{y} - \beta\,\overline{x} + \beta\,\overline{y} - \beta^2 z}{\alpha_0(y - x^3 - cx)}\,, \tag{17}$$

which proves that the origin has been stabilized in the $(\overline{x}, \overline{y}, z)$ coordinates. By considering (15) and (16), the following control law is obtained:

$$u = \frac{-y + z - \beta(x - y + z) - \beta^2 y}{\alpha_0(y - x^3 - cx)} \tag{18}$$

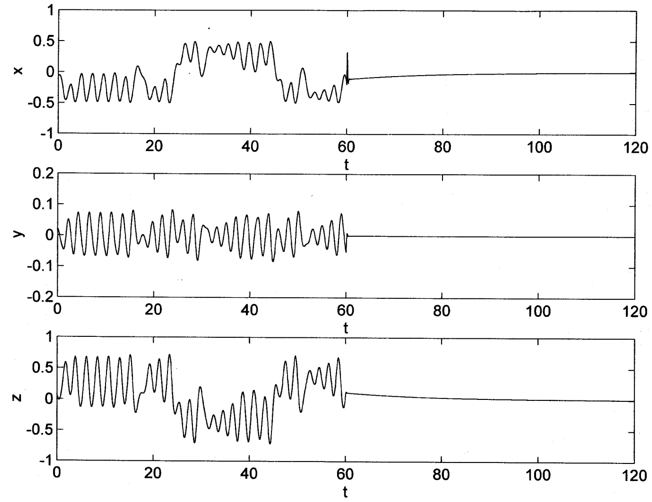


Fig. 11. Backstepping design applied to Chua's circuit: stabilization of the state variables x, y, and z, with the control u switched on at $t = 60$.

which enables the chaotic dynamics of the circuit to be stabilized at the origin in the (x, y, z) coordinates. The time waveforms of x, y and z are reported in Fig. 11. At first $u = 1$, then the control law (18) is switched on at $t = 60$.

5. Conclusion

In this Letter a Lyapunov-based approach, called backstepping design, has been proposed for controlling chaos. The advantages of the tool can be summarized as follows: (i) it is a *systematic* procedure for controlling chaotic or hyperchaotic dynamics; (ii) it can be applied to *several circuits and systems* reported in literature; (iii) both *stabilization* and *tracking* can be achieved, even if the target is outside the strange attractor. The technique has been successfully applied to the Lorenz system and Chua's circuit. Moreover, the robustness with respect to noise and the behavior in the presence of disturbance has been investigated.

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