An eigenvalue study of the MLC circuit.

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Abstract

The MLC circuit is the simplest non-autonomous chaotic circuit [01, 02, 03]. Insight in the behaviour of the circuit is obtained by means of a study of the eigenvalues of the linearized Jacobian of the non-linear differential equations [04]. The trajectories of the eigenvalues as functions of the parallel loss conductance are found. An explanation of the chaotic behaviour based on the behaviour of the autonomous system is given.

1. Introduction

The Murali-Lakshmanan-Chua circuit is composed of (a coil L with a series loss resistor R_s) in parallel with (a capacitor C with a nonlinear parallel loss conductor G_p) (Fig.1).

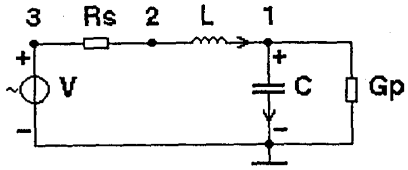

Fig. 1, The Murali-Lakshmanan-Chua circuit

The nonlinear loss conductor G_p may be realized by means of Chua's diode [05] (Fig. 2). By inserting an independent sinusoidal voltage source V in series with L and R_s chaos may be observed.

Because of only one nonlinear component G_p the trajectories of the eigenvalues of the linearized Jacobian of the nonlinear differential equations may be found by means of simple linear frequency analysis varying the dynamic value of G_p, gnl, in a specific case where L = 18mH, R_s = 1340Ω, C = 10nF. The dynamic value, gnl, of the nonlinear parallel conductor G_p is varied from -1e+19 to +1e+18.

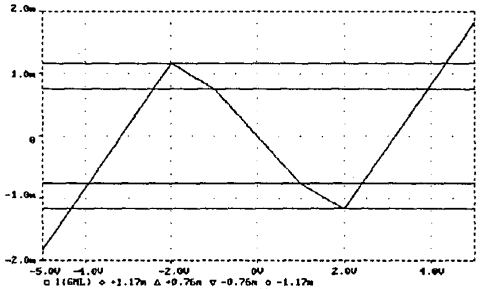

Fig. 2, The Chua diode characteristic, i = f(v)

2. Qualitative analysis

When the dynamic parallel loss conductance gnl is very large and positive the coupling between the coil L and the capacitor C is very small. The voltages and currents will become exponentially damped signals. The energy in connection with the coil L (the magnetic flux) will be transformed into heat in the resistor R_s with the time constant $\tau_L = L/R_s$ and the energy in connection with the capacitor C (the electric charge) will be transformed into heat in the conductor G_p with the time constant $\tau_C = C/G_p$. This behaviour corresponds to two real poles $s = p_1 = -R_s/L$ from the impedance $Z_L = R_s + s * L$ and $s = p_2 = -G_p/C$ from the admittance $Y_C = G_p + s * C$.

When gnl is very large and negative the signals will be exponentially increasing signals with the time constants mentioned above.

When the dynamic parallel loss conductance gnl is zero the circuit is a simple LC oscillator with series losses and the voltages and currents will be damped sinusoids. The circuit is a second order circuit. The eigenvalues of

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the linearized Jacobian are either two real poles or a pair of complex poles in the complex frequency plane. It is to be expected that the complex pair of poles will follow a trajectory which goes from one point to another on the real axis. The two points corresponds to two real double poles. The poles are the roots of the characteristic polynomial of the second order differential equation modelling the system:

$$s^2 + (2\alpha)\, s + \omega_0^2$$

where $2\alpha = \left(\frac{G_p}{C} + \frac{R_s}{L}\right)$ and $\omega_0^2 = \frac{1 + R_s G_p}{L\,C}$

The roots are:

$$p_1, p_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

For $\omega_0^2 > \alpha^2$ the roots become a pair of complex poles:

$$p_1, p_2 = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}.$$

For $\omega_0^2 = \alpha^2$ the roots become a pair of real double roots. The corresponding values of G_p becomes:

$$G_p = C\left(\frac{R_s}{L} \pm \frac{2}{\sqrt{L\,C}}\right) = \frac{C}{\tau_L} \pm 2\sqrt{\frac{C}{L}}$$

3. Quantitative Analysis.

By means of the formulas above Table 1 below is calculated for a specific set of parameters L = 18mH, R_s = 1340Ω and C = 10nF.

G_p	re1/re	re2/im
-1e+19	+1e+27	-74.444444e+3
-0.760000e-3	+10.918193e+3	-9.362627e+3
-0.74626760e-3	+112.21	+70.11
-0.74626754e-3	+91.15	+91.15
-0.74626750e-3	+91.15	±j 17.39
-0.41000000e-3	-16.7222222e+3	±j 47.15613e+3
0	-37.2222222e+3	±j 64.57602e+3
+2.23515640e-3	-148.980042e+3	±j 14.81
+2.23515643e-3	-148.980044e+3	-148.980043e+3
+2.23515650e-3	-149.900298e+3	-148.957115e+3
+1	-99.999944e+6	-74.500041e+3
+1e+18	-1e+26	-74.444444e+3

Table 1 Eigenvalues as functions of G_p

It is seen that for G_p = -0.74626754e-3 the real double pole s = +91.15 is in the right half plane. For G_p going to "minus infinite" the real pole re1 corresponding to the capacitor goes to "plus infinite" and the real pole re2 corresponding to the coil goes to -74.444e+3. Due to the maximum slope of -0.76mS for G_p in origo the maximum value for re1 becomes +10.92e+3. Due to this large real pole it is obvious that **the autonomous system has an unstable point of balance in origo.** Even very small initial conditions close to origo will give rise to exponentially increasing signals in positive or negative direction. If the autonomous system is started up with an initial condition of e.g. 1e-12 volt across the capacitor C the signals will increase exponentially until the bending point of the piecewise linear conductance G_p, i.e. it is to be expected that the voltage of C will rise to 1 volt when the complex pole pair for G_p = -0.41mS, s = $-16.72e+3 \pm j * 47.16e+3$, will take over and give rise to a damped oscillation. This behaviour is shown in Fig. 3 where the currents in the nonlinear conductance and the capacitor are shown as functions of time and of the voltage across.

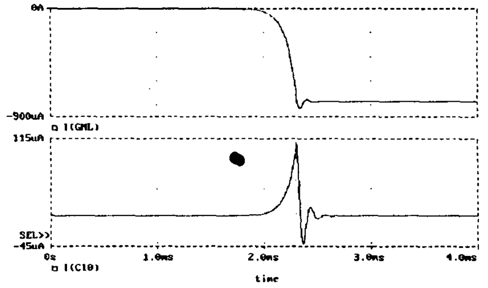

Fig.3a, Autonomous system performance.
i(gnl) and i(C10) as functions of time

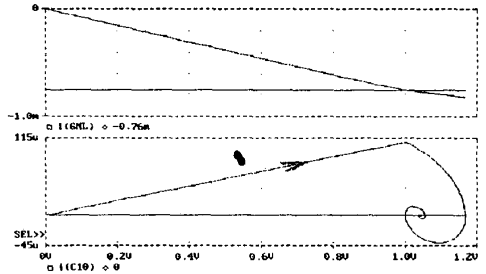

Fig.3b, Autonomous system performance.
i(gnl) and i(C10) as functions of V(1)

The trajectories of the eigenvalues are shown in the figures 4, 5 and 6. The dynamic value of the parallel conductance G_p, gnl, is varied from -1e+19 Siemens to +1e+18 Siemens. In Fig. 4 it is seen how the complex pole pair leaves the real axis for gnl = -0.746mS and

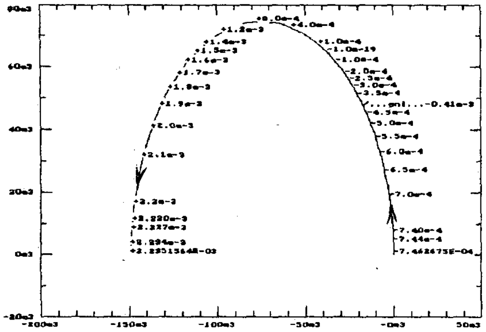

Fig. 4, Complex pole pair trajectory.
Positive imaginary part as function of real part.

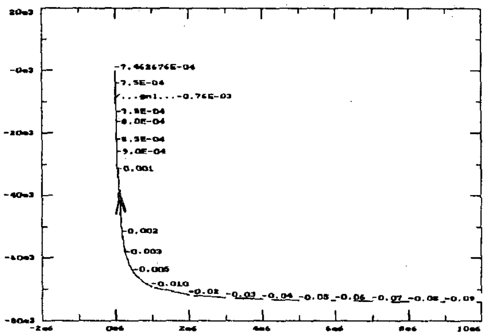

Fig. 5, Real poles "trajectory" for
negative values of gnl.

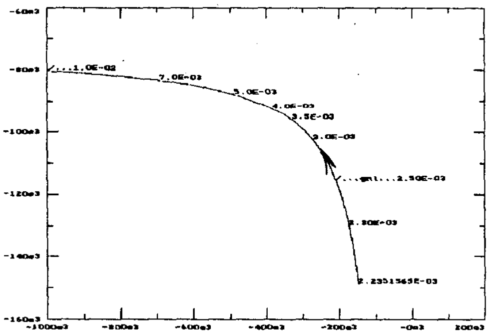

Fig. 6, Real poles "trajectory" for
positive values of gnl.

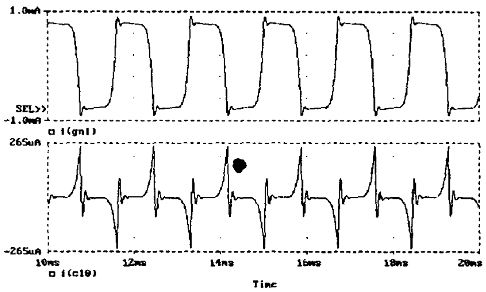

Fig. 7a, i(gnl) and i(C10) as functions of time.
Voltage source V:
Amplitude = 25mV, Frequency = 586.499016Hz

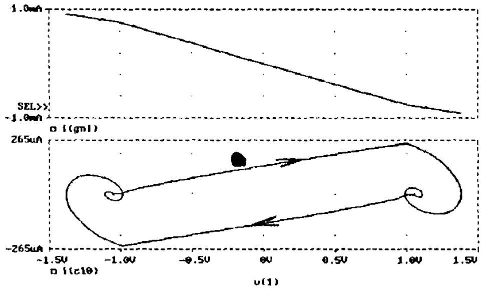

Fig. 7b, i(gnl) and i(C10) as functions of V(1)
Voltage source V:
Amplitude = 25mV, Frequency = 586.4990160Hz

returns back to the real axis for gnl = +2.235mS. The trajectory of the complex pole pair crosses the imaginary axis for gnl = -0.7444445E-03 where the real part of the complex pole becomes zero and the imaginary part becomes +3685.082000rps corresponding to the frequency 586.4990160Hz. In Fig. 5 the two real poles are pictured against each other for negative values of gnl. It is seen how the pole in connection with the coil goes to -74.4e+3 while the pole related to the capacitor goes to "plus infinite". In Fig. 6 the two real poles are pictured against each other for positive values of gnl. It is seen how the pole in connection with the capacitor goes to "minus infinite" while the pole related to the coil goes to -74.4e+3.

4. Limit Cycle and Chaotic Behaviour.

With knowledge about the eigenvalues of the system we may choose the frequency of the excitation deliberately in order to obtain limit cycle or chaotic behaviour when varying the amplitude of the independent voltage source. In the following PSpice with RELTOL=1e-6 is used for the simulations. In Fig. 7 and Fig. 8 the frequency 586.4990160Hz corresponding to the point where the trajectory crosses the imaginary axis is chosen. For an amplitude of 25mV (Fig. 7) it is seen how the currents i(gnl) and i(C10) are the same as in the autonomous case (Fig. 3). Due to the varying input voltage **a train of pulses** is obtained. Every time the current in the capacitor i(C10) becomes zero due to the pair of complex poles in the left halfplane the independent voltage source V will bring the circuit in a situation where the real pole in the right half plane occurs and a new pulse going to the other breaking point starts up.

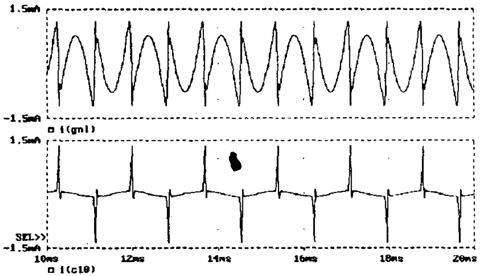

Fig. 8a, i(gnl) and i(C10) as funtions of time.
Voltage source V:
Amplitude = 5V, Frequency = 586.4990160Hz

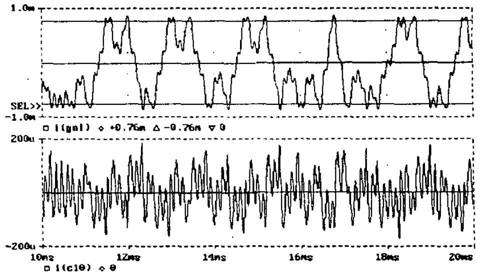

Fig. 9a, i(gnl) and i(C10) as functions of time.
Voltage source V:
Amplitude = 60mV, Frequency = 7505.13Hz

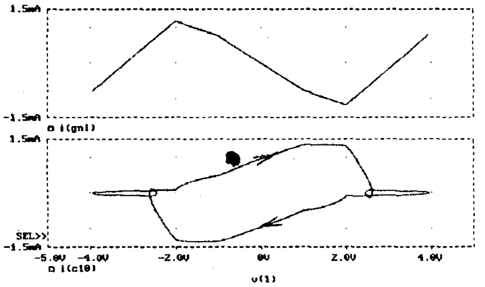

Fig. 8b, i(gnl) and i(C10) as functions of V(1)
Voltage source V:
Amplitude = 5V, Frequency = 586.4990160Hz

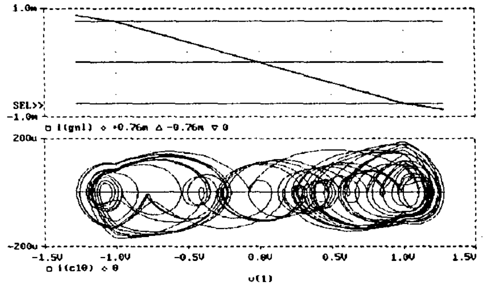

Fig. 9b, i(gnl) and i(C10) as functions of V(1).
Voltage source V:
Amplitude = 60mV, Frequency = 7505.13Hz

In Fig. 8 the amplitude of the independent voltage source is increased to 5V. It is seen how the damping of the current pulses in the capacitor becomes faster due to entering the areas with gnl = +1mS.

In the following the frequency is chosen to 7505.13Hz corresponding to gnl = -0.41mS. If the amplitude of the independent voltage source is 25mV a first order limit-cycle at one of the bending points is obtained. With increasing amplitude of V limit cycle and chaotic behaviour may be observed. For an amplitude of 54.9mV chaos around one of the bending points is found. For 55.00mV both bending points are involved in chaotic behaviour. For a time range of about 20ms chaos is around one bending point and then it changes to the other bending point for some time. At 55.50mV the intervals with chaos around one bending point becomes smaller. At 56.5mV we see a 3'rd order limit cycle around one of the bending points. In Fig. 9 the amplitude of V is 60mV and chaos including both bending points occur. At 100mV and 1V 1'st order limit cycles occur. For amplitude 1V it becomes necessary to change RELTOL to 1e-5 in PSpice in order to avoid problems with too small integration steps.

5. Conclusion.

The behaviour of the Murali-Lakshmanan-Chua circuit (MLC circuit) is investigated by means of a study of the eigenvalues of the linearized Jacobian of the non-liner differential equations. It is found that the autonomous circuit has an unstable point of balance in origo which give rise to chaotic behaviour in case of the non-autonomous circuit.

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