LETTER

Control of Chua's Circuit by Switching a Resistor

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SUMMARY In this letter a new method for controlling chaos is proposed. Although different several methods based on the OGY- and the OPF-method perturb a value of an accessible system parameter, the proposed method perturbs the only timing of switching three values of a parameter. We apply the proposed method to the well-known Chua's circuit on computer simulations. The chaotic orbits in the Rössler type- and the double scroll type-attractor can be stabilized on several unstable periodic orbits embedded within these attractors.

key words: *controlling chaos, Chua's circuit, unstable periodic orbits*

1. Introduction

Chaotic motions occurred in nature and man-made systems have been studied with academic interest. However, the presence of chaos in physical systems may be detrimental to the operation as it cannot be predicted. In recent years, to eliminate such unpredictable behavior, several methods for controlling chaotic motions have been proposed.

Ott, Grebogi, and Yorke (OGY) have suggested a method that takes advantage of a feature of chaos [1]-[4]. Their method (OGY method) is to convert the chaotic motions found in a physical system to periodic motions or aperiodicity by making only small time-dependent perturbations of an accessible system parameter. For low-dimensional chaos described by a 1-dimensional map, an occasional proportional feedback method (OPF method), which is essentially a limiting case of the OGY method, can be used for controlling chaotic motions [5], [6]. Many chaotic physical systems have been controlled by the OGY- and the OPF-method [7]-[9].

For time-continuous chaotic systems, these methods slightly perturb the value of the parameter periodically. In some chaotic systems (e.g., certain of chemical and biological systems), it is not allowed that one slightly perturbs the value of the parameter. These methods are useless to such systems. In this letter, we propose a new method that perturbs the only timing of switching a parameter. The switched parameter employs the three closed values. Since the control systems with the proposed method need the three values of the parameter, we have two advantages. First, one may control the above uncontrolled chaotic systems. Secondly, the control systems may be simple structure. The proposed method is meant for controlling low-dimensional chaotic systems described by 1-D maps. Our computer simulations show that the method can be applied successfully to the well-known Chua's circuit.

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2. Control of Chua's Circuit

The dynamics of Chua's circuit [10] is given by

$$\begin{cases} dx/dt = G_0\cdot(y-x)/C_1 - g(x)/C_1 \\ dy/dt = G_0\cdot(x-y)/C_2 + z/C_2 \\ dz/dt = -y/L \\ g(u) = m_0 u + \frac{1}{2}(m_1-m_0)[|u+B_P|-|u-B_P|], \end{cases} \tag{1}$$

where x, y, z are the voltage across the capacitor C_1, the voltage across the capacitor C_2, and the current through the inductor L, respectively. There are three unstable fixed points $X_p={}^t(x_p, 0, z_p)$, $X_0={}^t(0, 0, 0)$, $X_m={}^t(x_m, 0, z_m)$,

$$\begin{aligned} x_p &= B_p(m_1-m_0)/(m_0+G_0), \\ z_p &= G_0 B_p(m_0-m_1)/(m_0+G_0), \\ x_m &= B_p(m_0-m_1)/(m_0+G_0), \\ z_m &= G_0 B_p(m_1-m_0)/(m_0+G_0). \end{aligned} \tag{2}$$

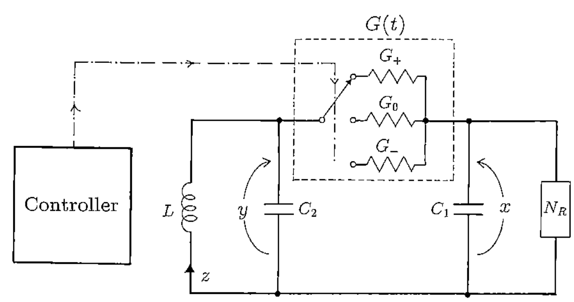


Fig. 1 Chua's circuit system to be controlled.

A Chua's circuit system to be controlled is shown in Fig. 1. Two resistors $G_+ = G_0 + G_*$, $G_- = G_0 - G_*$ are connected in parallel to the original resistor G_0. We assume that a value of G_* is sufficiently small. The controlled system shown in Fig. 1 is given by

$$\begin{cases} dx/dt = G(t)\cdot(y-x)/C_1 - g(x)/C_1 \\ dy/dt = G(t)\cdot(x-y)/C_2 + z/C_2 \\ dz/dt = -y/L \\ g(u) = m_0 u + \frac{1}{2}(m_1 - m_0)[|u+B_P| - |u-B_P|], \end{cases} \tag{3}$$

where $G(t)$ employs the only three closed values (G_+, G_0, G_-). The chaotic orbit strikes the Poincaré section $\Sigma_+ = \{x, y, z | z = z_p, y < 0\}$ or $\Sigma_- = \{x, y, z | z = z_m, y > 0\}$ at a discrete time $t_j (j = 1, 2, \cdots)$ (see Fig. 2). Period T of UPO($\phi_u(t) = \phi_u(t+T)$) we want to stabilize can be described by

$$T = T_1 + T_2 + \cdots + T_j + \cdots + T_i,$$
$$T_j = t_{j+1} - t_j, \tag{4}$$

where i indicates the number of times that UPO strikes the sections. The controller selects an appropriate resistor (G_+ or G_-) at time t_j. It holds the selected resistor for some period of time and then returns to the resistor G_0. We introduce a value τ_j that governs the controller. The value τ_j is calculated at time t_j. The value τ_j is used to determine the resistor values (G_+, G_-) and the holding time. The resistor $G(t)$ is selected as follows,

$$G(t) = \begin{cases} G_+ & (t_j < t < t_j + |\tau_j|,\ \tau_j > 0) \\ G_0 & (t_j + |\tau_j| < t < t_j + T_j) \\ G_- & (t_j < t < t_j + |\tau_j|,\ \tau_j < 0). \end{cases} \tag{5}$$

If $\tau_j > 0$, then the controller selects G_+ and holds for $|\tau_j|$ period of time. On the other hand, if $\tau_j < 0$, then G_- is selected and held for $|\tau_j|$ period of time (Fig. 3).

The dynamics on the Poincaré sections is given by

$$\boldsymbol{\xi}_{j+1} = P(\boldsymbol{\xi}_j, \tau_j),$$
$$\boldsymbol{\xi}_j = {}^t(x_j, y_j, z_j) = {}^t(x(t_j), y(t_j), z(t_j)), \tag{6}$$

where z_j is z_p or z_m. Above equation can be approximately rewritten as

$$y_{j+1} = P_y(y_j, z_j, \tau_j),$$
$$z_{j+1} = P_z(y_j, z_j, \tau_j), \tag{7}$$

since the chaotic attractors of the circuit on Σ_+ and Σ_- can be regarded as a line. We assume that τ_j is determined by the following function H_j.

$$\tau_j = H_j(y_{fj} - y_j),\ y_{fj} = \{\phi_u(t_j)\}_y. \tag{8}$$

The selecting the values of the resistor and the determining the period of time, therefore, are equivalent to the determining the function H_j. If the following conditions are satisfied, the chaotic orbits converge on UPO$\phi_u(t)$.

$$|y_{f(j+1)} - y_{(j+1)}| \leqq |y_{fj} - y_j|. \tag{9}$$

Since it is difficult to determine theoretically the function H_j that satisfies the above condition, we calculate a set A_j.

$$A_j = \{(y_{fj} - y_j), \tau_j | -T_j < \tau_j < T_j, |y_{fj} - y_j| < \delta y,$$
$$|y_{f(j+1)} - y_{(j+1)}| \leqq |y_{fj} - y_j|\} \tag{10}$$

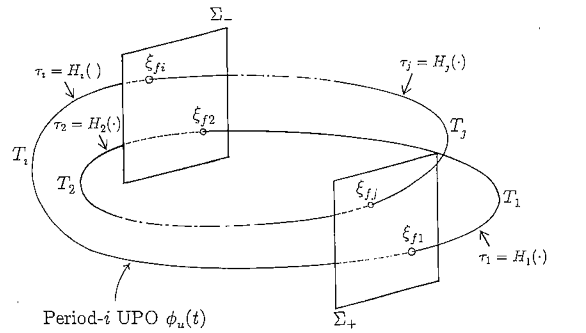


Fig. 2 The period-i UPO $\phi_u(t)$ and the Poincaré sections Σ_+, Σ_-.

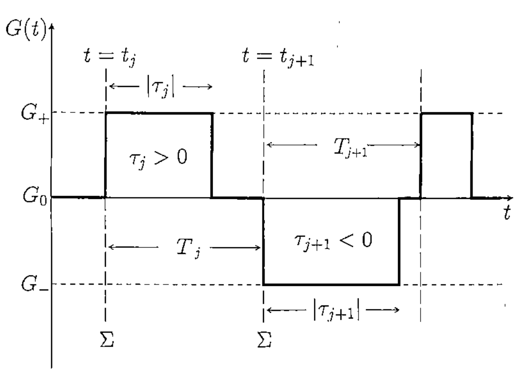


Fig. 3 The resistor $G(t)$ of the circuit vs. time t.

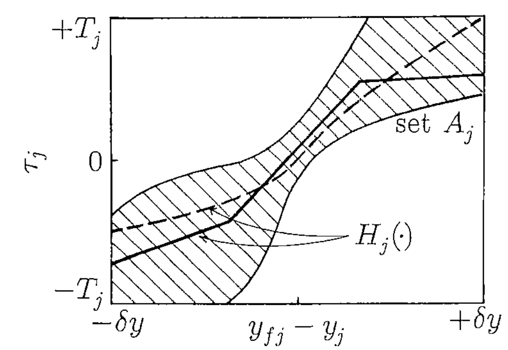
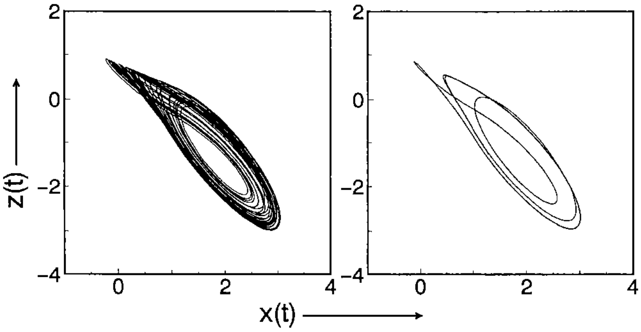


Fig. 4 The relationship between the set A_j and the function H_j. The function $\tau_j = H_j(y_{fj} - y_j)$ must be included in the set A_j to satisfied the condition of Eq. (9).



(a) (b)

Fig. 5 (a) The Rössler type attractor without control. (b) A stabilized period-3 UPO. (G_0=0.657, G_*=0.002, δy=0.01).

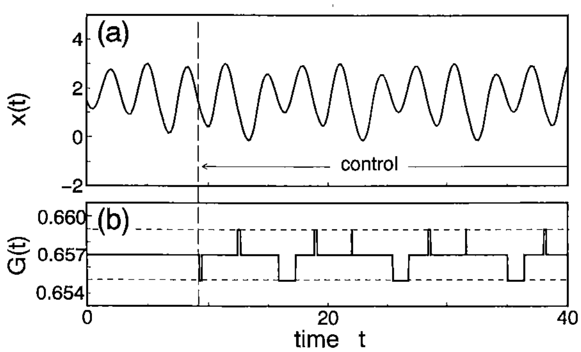
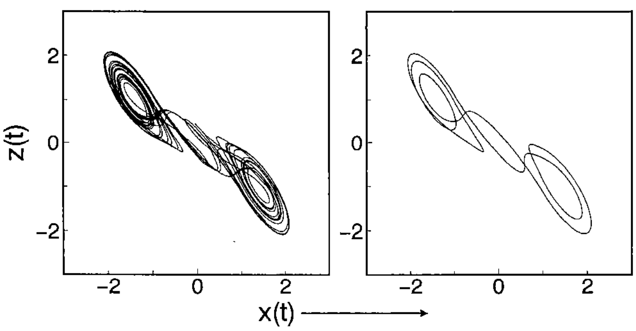


Fig. 6 The waveform of the variable $x(t)$ and the resistor $G(t)$ for stabilization of the period-3 UPO.



(a) (b)

Fig. 7 (a) The double scroll type attractor without control. (b) A stabilized period-5 UPO. (G_0=0.710, G_*=0.002, δy= 0.01).

The chaotic motions can be stabilized with any function H_j included in the set A_j as shown in Fig. 4.

Numerical simulation results are described. Figure 5 shows the Rössler type attractor (G_0=0.657) without control and a stabilized period-3 UPO. The waveform of the variable $x(t)$ and the resistor $G(t)$ of the controlled circuit system are shown in Fig. 6. The variable $x(t)$ exhibits a chaotic motion until t= 9.2715. At $t = 9.2715$, the switching of the resistor $G(t)$ starts, and then the variable $x(t)$ is successfully stabilized on the period-3 UPO. Figure 7 shows the double scroll type attractor (G_0=0.710) without control and a stabilized period-5 UPO. Figure 8 indicates the waveform of the variable $x(t)$ and the resistor $G(t)$. The variable $x(t)$ exhibits the double scroll attractor until t=160.0925. At t=160.0925, the switching starts, and then the stabilizing UPO is successfully implemented.

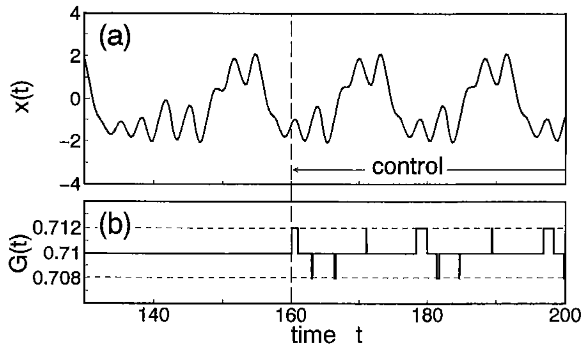


Fig. 8 The waveform of the variable $x(t)$ and the resistor $G(t)$ for stabilization of the period-5 UPO.

3. Conclusions

In this letter we have proposed a new control method that switches three values of a parameter. It has been verified that the method can be applied to Chua's circuit by using the resistor as the switched parameter. The method switches the values of the parameter at each time the orbit strikes the section Σ_+ or Σ_-, so that high-period UPO's can be stabilized. If the controller operates at periodic intervals T such as the OGY method, the structure of the set A_j is too complicated to determine the function H_j for high-period UPO's. It could stabilize the only low-period (period-1, -2) UPO's [11], and could not stabilize UPO's distributed in both regions $\{x, y, z|x > B_p\}$ and $\{x, y, z|x < -B_p\}$. Many UPO's, however, can be stabilized with use of the proposed method.

When the accurate chaotic dynamics is unknown, the proposed method cannot be used for control chaos. We intend to employ neural networks to control chaos under such unknown situations. Using neural networks, we think that the function H_j can be determined without the set A_j.

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