

LETTER TO THE EDITOR

Experimental observation of riddled behaviour

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Abstract. We observed riddled behaviour in a nonlinear electronic system consisting of a pair of mutually coupled Chua circuits. Our results show that there are real experimental systems for which repetition of results cannot be guaranteed.

There has recently been considerable study of chaotic synchronized systems [1-3]. In a simple synchronization procedure [2, 3] two identical n -dimensional chaotic systems $\dot{x} = f(x)$ and $\dot{y} = f(y)$ are coupled unidirectionally. This coupling creates a $2n$ -dimensional augmented dynamical system

$$\dot{x} = f(x) + D(y - x) \quad \dot{y} = f(y) \quad (1)$$

where $x, y \in \mathbb{R}^n$, $n \geq 3$, $D \in \mathbb{R}$. The full phase space of system (1) possesses a smooth invariant n -dimensional manifold M defined by the synchronized state $x = y$, i.e. any orbit originating in this manifold stays there forever approaching chaotic attractor A of dynamical system $\dot{x} = f(x)$. It has been shown [2, 3] that there exists such a value of $D = D_1$ that for $D > D_1$ two chaotic systems can synchronize for all slightly different initial values of x and y . In this case the chaotic attractor A of an n -dimensional dynamical system is asymptotically stable in $2n$ -dimensional phase space of the augmented system (1) [4]. For smaller values of D the chaotic attractor A can still be stable, i.e. the $2n$ -dimensional basin $b(A)$ with positive Lebesgue measure exists but $b(A)$ does not contain the neighbourhood of A . In this region the chaotic attractor A can have a locally riddled basin if there is an $\varepsilon > 0$ such that for every point $x \in b(A)$ any arbitrarily small ball centred on x contains a positive measure set of points with orbits that exceed the distance ε from A . Generally the spectrum of Lyapunov exponents of system (1) can be divided into two subsets $\lambda^{(1)}$ tangential and $\lambda^{(2)}$ transverse to the invariant manifold. It has been shown that if all transverse Lyapunov exponents are negative then chaotic attractor A is stable [1]. If one of the transverse Lyapunov exponents becomes positive (for example for $D = D_0$) we observe a blow-out bifurcation (a chaos-hyperchaos transition) [4] in which attractor A loses its stability. Riddled basins in the interval $[D_0, D_1]$ have been observed numerically in several systems [5]. Systems with a riddled basin have the property that it is impossible to predict from given initial conditions what trajectory in the phase space the system will follow. Any two neighbouring initial conditions may lead to wildly divergent trajectories approaching different attractors. The level of uncertainty introduced by riddled

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behaviour is much greater than that associated with chaotic behaviour where, due to the sensitive dependence on initial conditions, two nearby trajectories diverge exponentially but evolve on the same strange attractor.

In this letter we report the first known experimental observation of riddled behaviour; the experimental system is a nonlinear electronic circuit operating in the neighbourhood of a blow-out bifurcation.

We consider a system consisting of two unidirectionally coupled identical Chua circuits [6] as shown in figure 1. Each circuit contains three linear energy-storage elements (an inductor and two capacitors), a linear resistor and a single nonlinear resistor N_R , with a three-segment piecewise-linear v - i characteristic defined by

$$f(v_R) = m_0 v_0 + \frac{1}{2}(m_1 - m_0)[|v_R + B_p| - |v_R - B_p|]. \quad (2)$$

This relation is shown graphically in figure 2, the slopes in the outer and inner regions are m_0 and m_1 respectively, $\pm B_p$ denotes the breakpoints.

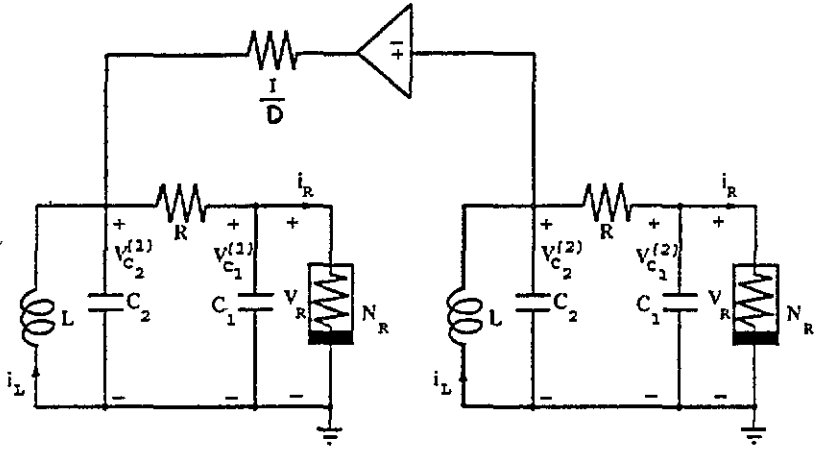


Figure 1. Two identical unidirectionally coupled Chua circuits.

The state equations for the circuit in figure 1 are as follows.

$$\begin{aligned} C_1 \frac{dv_{C_1}^{(1)}}{dt} &= G(v_{C_2}^{(1)} - v_{C_1}^{(1)}) - f(v_{C_1}^{(1)}) \\ C_2 \frac{dv_{C_2}^{(1)}}{dt} &= G(v_{C_1}^{(1)} - v_{C_2}^{(1)}) + i_L^{(1)} + D(v_{C_2}^{(2)} - v_{C_2}^{(1)}) \\ L \frac{di_L^{(1)}}{dt} &= -v_{C_2}^{(1)} \\ C_1 \frac{dv_{C_1}^{(2)}}{dt} &= G(v_{C_2}^{(2)} - v_{C_1}^{(2)}) - f(v_{C_1}^{(2)}) \\ C_2 \frac{dv_{C_2}^{(2)}}{dt} &= G(v_{C_1}^{(2)} - v_{C_2}^{(2)}) + i_L^{(2)} \\ L \frac{di_L^{(2)}}{dt} &= -v_{C_2}^{(2)} \end{aligned} \quad (3)$$

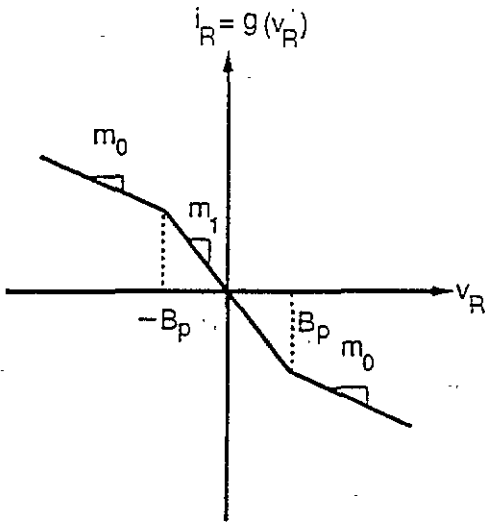


Figure 2. v - i characteristic of a nonlinear resistor.

where $G = 1/R$. It is well known that for $C_1 = 10$ nF, $C_2 = 99.34$ nF, $m_1 = -0.76$ mS, $m_0 = -0.41$ mS, $L = 18.46$ mH, $R = 1.64$ k Ω and $D = 1/R_c = 0$, both circuits operate on the chaotic double-schroll attractor [6]. If $x(0)$ is slightly different from $y(0)$ ($x = [v_{C_1}^{(1)}, v_{C_2}^{(1)}, i_L^{(1)}]^T$, $y = [v_{C_1}^{(2)}, v_{C_2}^{(2)}, i_L^{(2)}]^T$) the two trajectories $x(t)$ and $y(t)$ diverge exponentially from each other. Depending on the value of D which we consider as a control parameter, synchronization can be achieved if we add the perturbation signal

$$D(v_{C_2}^{(2)} - v_{C_2}^{(1)}) \tag{4}$$

to the first circuit [2,3]. As shown in [3] this method of synchronization is very useful for experimental systems as, due to continuous control, perturbation signal (4) is corrected automatically when noise is present in the experiments.

As shown in [3] blow-out bifurcation (a chaos-hyperchaos transition) for system (3) occurs at the point $D = 1.17$. For $D > 1.17$ the typical trajectory of (3) is characterized by two positive Lyapunov exponents (one of the transverse Lyapunov exponents is positive) and evolves on hyperchaotic attractor with Lyapunov dimension $d_L > 4$. For D slightly larger than 1.17, for a typical trajectory on the attractor all transverse Lyapunov exponents are necessarily negative, but there are still initial conditions dense in the attractor for which one of the transverse exponents is positive. For these values of D we can expect riddled basins of two attractors: a chaotic attractor A in the case of synchronization and a hyperchaotic attractor in the case of no synchronization.

In our experiments we set the value of $D = 1/R_c = 1.18$ and observed 1000 independent experiments analysing $v_{C_1}^{(1)}$ against $v_{C_1}^{(2)}$ plots on the oscilloscope. Such plots represent two-dimensional projections of the attractor and, in the case of synchronization, give a straight line. As there was no possibility of controlling the initial conditions of both circuits, $x(0)$ and $y(0)$, we assumed that, after each experiment when our system was switched off, new initial conditions were established at random when the system was switched on again. For the 1000 experiments we observed 381 in which both circuits synchronized (figure 3(a)) and 619 in which the hyperchaotic attractor shown in figure 3(b) was created. We repeated our experiments for D equal to 1.185 and 1.190. With a larger distance from the blow-out bifurcation (the chaos-hyperchaos transition) point, synchronization became more probable as we observed 451 and 734 experiments, respectively, with this result.

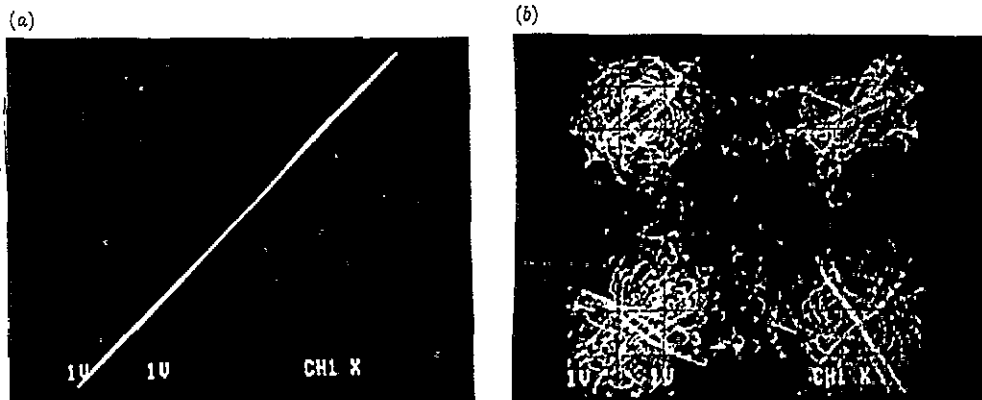


Figure 3. Experimental $v_{C_1}^{(1)}$ against $v_{C_1}^{(2)}$ plots of unidirectionally coupled Chua circuits for $K = 1.18$: the horizontal axis is $v_{C_1}^{(1)}$, 1 V/div, the vertical axis is $v_{C_1}^{(2)}$, 1 V/div; (a) synchronization (chaotic attractor A) and (b) no synchronization (hyperchaotic attractor).

Before each experiment it was impossible to predict the result based on previous ones, i.e. repetition of previous results could not be guaranteed, so we can say that we observed riddled behaviour.

These results demonstrate that riddled behaviour is an experimentally observable phenomenon. We expect this phenomenon to occur in higher-dimensional nonlinear systems operating close to the blow-out bifurcation (the chaos-hyperchaos transition).

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