EXPERIMENTAL STUDY OF IMPULSIVE SYNCHRONIZATION

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ABSTRACT

In this paper, the impulsive synchronization is examined by using two kinds of experimental circuits, that is, Chua's oscillators and hyperchaotic circuits. Furthermore, its performance was verified by constructing typical chaos-based communication systems.

1. INTRODUCTION

Over past ten years, a number of interesting spread spectrum communication systems are proposed [1]–[6], which utilize the synchronized chaotic systems. In these systems, perfect synchronization is usually expected to recover the informational signal. That is, the recovery of the information signal requires the receiver's own copy of the chaotic spreading sequence which is synchronized with the transmitter's one. Therefore, the synchronization is a key requirement in the design of chaos-based spread spectrum communication systems.

Recently, a new control method, that is, impulsive synchronization is developed, which allows the synchronization of chaotic systems using only small pulses [5]. This method is applied to several chaos-based communication systems, and they exhibit good performance for the synchronization requirement [4], [5], [6]. In this scheme, the transmitted signal consists of a sequence of time frames. Every frame has a length of T seconds and consists of two regions. The first region of the time frame is a synchronization region consisting of synchronization pulses. The synchronization impulses are used to impulsively synchronize the chaotic systems in both transmitter and receiver. The second region is the scrambled signal region where the scrambled signal is contained. Within every time frame, the synchronization region has a length of Q ($< T$) and the remaining time interval $T - Q$ is the scrambled signal region. Since Q is usually very small compared with T, the lost of time for packing message signal is negligible [5].

Judging from the computer studies of [5], [6], the impulsive synchronization method seems to be a promising one. However, from the experimental viewpoint, it is not so easy to synchronize two laboratory circuits by using the impulsive synchronization. It is due to the following reason:

- In the laboratory circuits, noise is unavoidable.
- Some parameter mismatch and change (drift) of the chaotic circuits between the transmitter and the receivers are unavoidable.
- The synchronization region Q cannot be chosen to be sufficiently small, especially for hyperchaotic circuits, because the noise and the parameter mismatch will soon desynchronize the chaotic circuits even if the two circuits have synchronized at a moment by the impulsive synchronization.

The first experimental results on the impulsive synchronization were presented in [7]. Two Chua's oscillators are effectively synchronized by using narrow impulses ($Q/T = 0.16\%$, $1/T = 18000$). In order to apply the impulsive synchronization to the chaotic communication systems, we need to make more detailed studies of impulsive synchronization and its performance analysis, that is,

- Examination of the impulsive synchronization for more wide frequency range and more noisy channels.
- Evaluation of the minimum length of Q and the ratio of Q to T.
- Application of the impulsive synchronization to hyper chaotic circuits or some other circuits.

In this paper, we analyze the performance of the impulsive synchronization by building two kinds of chaotic circuits, that is, Chua's oscillator and the hyperchaotic circuit [8]. First, we synchronize them by using impulsive synchronization via one communication channel. Next, we evaluate the minimum length of the interval Q which gives the perfect synchronization, and the ratio of Q to T. Furthermore, we verify its performance by using some communication systems. That is, we construct the chaos-based spread spectrum communication systems by using Chua's oscillators and the hyperchaotic circuits.

2. EXPERIMENTAL RESULTS

In this section, we discuss the experimental results of the impulsive synchronization. First, we build two kinds of chaotic circuits whose parameter mismatch is within 0.5%. Then, they were synchronized by using the impulsive synchronization. Next, we evaluate the minimum length of Q and the ratio of Q to T. Furthermore, we add small noise to the drive signal and examined the robustness of the impulsive synchronization to additive noise.

2.1. Chua's Oscillator

The dynamics of Chua's oscillator is given by

$$C_1 \frac{dv_1}{dt} = \frac{v_2 - v_1}{R} - f(v_1), \quad (1)$$

$$C_2 \frac{dv_2}{dt} = \frac{v_1 - v_2}{R} + i, \quad (2)$$

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$$L\frac{di}{dt} = -v_2 - ri, \quad (3)$$

where $f(\cdot)$ is the nonlinear characteristic of Chua's diode, defined by $f(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)(|\, v_1 + E \,| - |\, v_1 - E \,|)$ and B_p is the breakpoint voltage of Chua's diode. The parameters are given by

$$C_1 = 10.3[nF], C_2 = 99.6[nF], L = 22.3[mH],$$
$$R = 885[\Omega], r = 68.9[\Omega], B_p = 0.78[V],$$
$$G_b = -0.516[mS], G_a = -0.875[mS].$$

We observed the case where the voltage v_1 is the drive signal. Figures 1-2 show the minimum length of the interval Q which gives the *perfect synchronization* [1] and the ratio of Q to T, respectively. We get the following results from the experimental study.

(1) For the case: $T \leq 9.0 \times 10^{-6}$

The perfect synchronization requires only 8% of the frame region. That is, 92% of the frame region is available for transmitting scrambled signals. In this case, the minimum length of Q increases in proportion to the length of T. Therefore, we can choose the length of Q sufficiently small compared with T as shown in [4], [5], [6].

(2) For the case: $5.0 \times 10^{-3} \leq T \leq 9.0 \times 10^{-6}$

The ratio of Q to T rises from 8% to 96% as T is increased. Thus, the available region for transmitting scrambled signals decreases.

Next, we added small noise to the drive signal, and examined the robustness of the impulsive synchronization to additive noise. There is no great difference between the impulsive synchronization and the continuous synchronization. That is, the thickness of the oscilloscope trace of synchronized states is almost same. Thus, the impulsive synchronization seems to be robust for small additive noise.

2.2. Hyperchaotic Circuit

We applied the impulsive synchronization method to the hyperchaotic circuit [8]. The dynamics of this circuit is given by

$$C_1\frac{dv_1}{dt} = g(v_2 - v_1) - i_1, \quad (4)$$
$$C_2\frac{dv_2}{dt} = -g(v_2 - v_1) - i_2, \quad (5)$$
$$L_1\frac{di_1}{dt} = v_1 - r_1 i_1 + R i_1, \quad (6)$$
$$L_2\frac{di_2}{dt} = v_2 - r_2 i_2, \quad (7)$$

where $g(\cdot)$ is the v-i characteristic of Chua's diode, defined by $g(v_2 - v_1) = G_a(v_2 - v_1) + \frac{1}{2}(G_b - G_a)(|\, v_2 - v_1 - B_p \,| - |\, v_2 - v_1 + B_p \,|)$, and B_p is the breakpoint voltage of Chua's diode. The parameters are given by

$$C_1 = 102[nF], \; C_2 = 318[nF], \; L_1 = 29.9[mH],$$
$$L_2 = 22.3[mH], \; R = 330[\Omega], \; r_1 = 71.2[\Omega],$$
$$r_2 = 42.9[\Omega], \; G_a = 31.6[mS], \; G_b = -0.667[mS],$$
$$B_p = 0.50[V].$$

These parameters are different from the original parameters used in [8] because we have some difficulty in building the circuit with the same elements.

We synchronize the hyperchaotic circuits by using one communication channel. In this case, every frame consists of three regions. The first and the second regions of the time frame are synchronization regions consisting of driving signals. That is, the first region with a length of Q is used to transmit the signal $v_2 - v_1$, and the second region with a length of Q is used to transmit the signal $-Ri$. The third region with a length of $T - 2Q$ is usually used to transmit the scrambled signals. In this experimental study, we do not use the third region. By using this method, we can synchronize the two hyperchaotic circuits. Figures 3-4 illustrate the minimum length of Q and the ratio of Q to T, respectively. From the experimental study, we found the following results.

(1) For the case: $3.8 \times 10^{-6} \leq T \leq 1.0 \times 10^{-5}$

The ratio of Q to T is equal to 26%. Thus, the perfect synchronization requires at least 52% ($= \frac{2Q}{T} = 26\% \times 2$) of the frame region. That is, 48% of the frame region is available for transmitting the information signals. In this case, the minimum length of Q increases in proportion to the length of T.

(2) For the case: $1.3 \times 10^{-5} \leq T \leq 2.0 \times 10^{-5}$

The perfect synchronization requires 100% ($= 50\% \times 2$) of the frame region. Therefore, no region is available for transmitting information signals.

Next, we examined the robustness of the impulsive synchronization to additive small noise, and found that the impulsive synchronization for the hyper chaotic circuit seems to be robust for small additive noise.

3. SPREAD SPECTRUM COMMUNICATION SYSTEMS

We can easily realize the chaotic masking system [2] by using the impulsive synchronization method (though the chaotic masking is not secure enough). That is, the synchronization region Q of the transmitted signal consists of synchronization pulses, and the scrambled region $T - Q$ consists of the signal $p(t) = s(t) + c(t)$. The recovery of the information signal $s(t)$ is performed by subtracting $\tilde{c}(t)$ from $p(t)$ ($\tilde{c}(t)$ is the carrier signal, which is synchronized with the transmitter's one at the receiving side). Therefore, the recovering process is a little simpler than the original one [2]. Similarly, the direct sequence spread spectrum is easily realized by multiplying the chaotic signal $c(t)$ by the information signal $s(t)$ [4].

In those communication systems, the lost of time for packing message signal is not negligible since Q is not al-

[1] We observed the oscilloscope trace of the synchronized states (lissajou), that is, the x-axis traces the voltage v_2 of the drive system, and the y-axis traces the voltage v_2 of the slave system, which usually draws 45-degree line as shown in [7]. The perfect synchronization means the following state:
(1) The oscilloscope trace draws a 45-degree line, which is sufficiently thin.
(2) No disturbance is found in a synchronized 45-degree line.
Thus, the experimental results greatly depend on both the measuring instruments and the circuit elements.

ways small compared with T. As the results, the recovered signal $r(t)$ has the waveform which is periodically sampled every T seconds. Thus, the information signal $s(t)$ is expected to be bandlimited to the frequency range below $\frac{1}{2T}$ according to the sampling theorem.

We constructed the above spread spectrum communication systems by using Chua's oscillators and hyperchaotic circuits. These communication systems work well, and we can get the recovered signal which is sampled every T seconds (See Figs. 5-6). Therefore, we conclude that the impulsive-synchronization is useful for the purpose of synchronizing chaotic systems in the transmitter and receiver.

4. CONCLUSION

We have examined the impulsive synchronization method by using the experimental circuits. Furthermore, its performance was verified by constructing some communication systems. We concluded that the impulsive synchronization is useful for the purpose of synchronizing chaotic systems. However, the recovered signal has the waveform which is periodically sampled every T seconds because Q is not small enough. Therefore, if we can make the length of the synchronization region Q much shorter (by using high accurate elements), then the communication systems will exhibit higher performance.

5. REFERENCES

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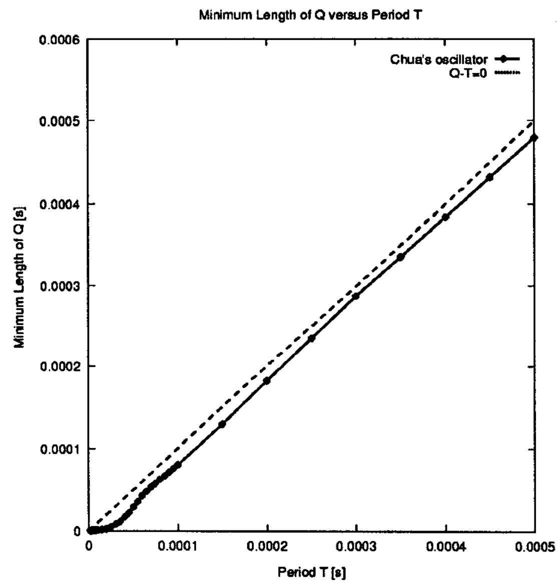

Figure 1: Minimum length of Q versus frame length T for Chua's oscillator.

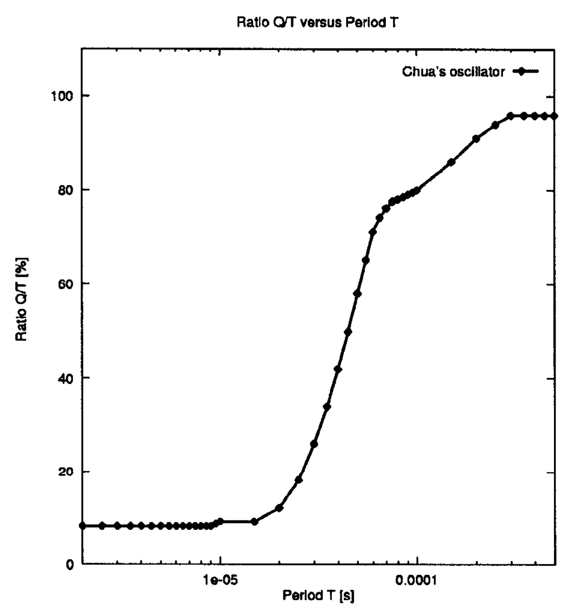

Figure 2: Ratio of Q to T versus frame length T for Chua's oscillator.

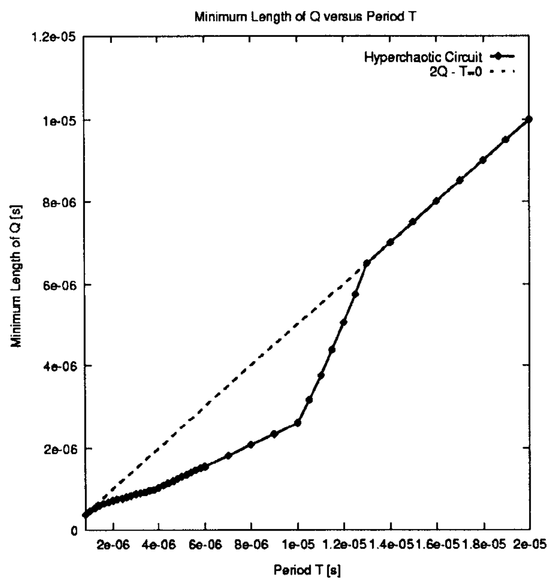

Figure 3: Minimum length of Q versus frame length T for the hyperchaotic circuit.

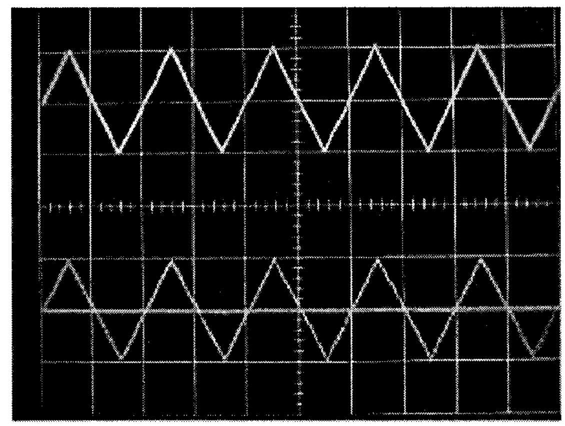

Figure 5: Information signal versus recovered signal. Chua's oscillator and the chaotic masking are used as a chaotic carrier generator and a chaotic modulation, respectively. ($T = 2.0 \times 10^{-5}$, $\frac{Q}{T} = 0.50$)

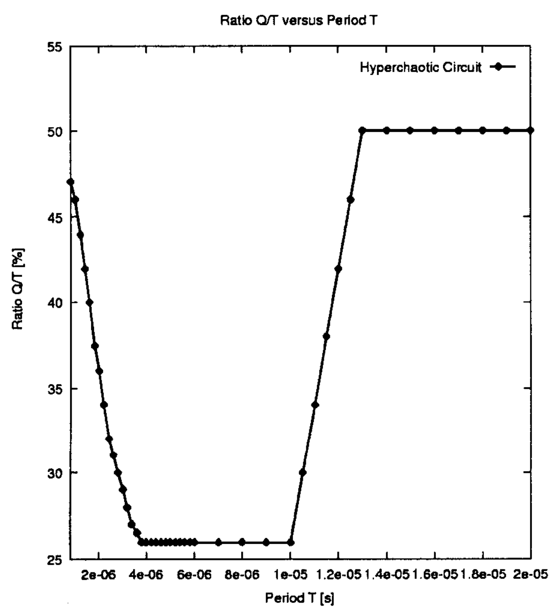

Figure 4: Ratio of Q to T versus frame length T for the hyperchaotic circuit.

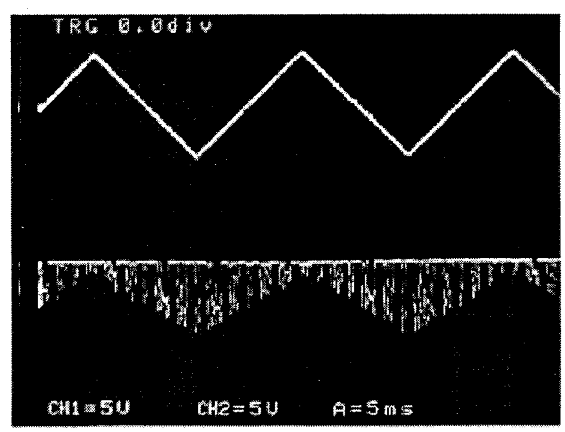

Figure 6: Information signal versus recovered signal. The hyperchaotic circuit and the direct sequence are used as a chaotic carrier generator and a chaotic modulation, respectively. ($T = 2.5 \times 10^{-6}$, $\frac{2Q}{T} = 0.68$)