Signal Transmission through a Chain of Chua's circuits

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ABSTRACT

In this paper, the possibility of signal transmission through a chain of Chua's circuits, each one operating in a chaotic regime, is investigated. The propagation of an analog signal through a nearly synchronized chain is studied. As we increase the linear coupling resistance between cells, the chaotic noise is found to increase and propagate through the cells. This phenomenon is exploited in a new method for transmitting digital signals with a noise reduction circuit. We also present results on signal transmission delay. One of the most interesting results reported is our demonstration that it is possible under certain conditions to recover a binary signal even if no synchronization exists between cells.

I. INTRODUCTION

Numerous experimental and theoretical investigations have been carried out to study the chaotic behavior and the bifurcation phenomena of Chua's circuit. Presently, a great deal of interests have been focused on applications of *chains* and *arrays* of Chua's circuits [1]. Here, a *chain* (resp., *array*) is a special case of a one-dimensional (resp., two-dimensional) *cellular neural network* having a particular chain of "cells" and templates [2].

The synchronization property of a chain of Chua's circuits was first studied by V. Perez-Munuzuri et al. [3]. Its dynamic phenomena was investigated by A.M. Dąbrowski et al., but they did not include a driving signal [4]. Numerical examples of a traveling wave front in the chain were given by V. Perez-Munuzuri et al. [5]. Recently, Güzeliş proposed a chaotic neural network made of Chua's circuits [6]. However, its chaotic behavior was not investigated fully. In order to study chaotic neural networks, biological systems, and physical systems, we need to investigate the transmission property of digital signals, since the information signals are sometimes carried by pulse sequences.

In this paper, we study a chain of Chua's circuits driven by a *current source.* We shall first investigate the propagation property of an *analog* signal by a chain. In this case, the cells (identical Chua's circuit) are operating in a chaotic regime but are *nearly synchronized.* As the resistance of the linear resistor which coupled the cells increases, the "chaotic" noise increases and propagates through the cells. Consequently, the information signal being transmitted is corrupted by the chaotic noise. To exploit this phenomenon, a new method for transmitting a *digital* signal through a *disordered* chain is proposed. Furthermore, a comparator and a noise-reduction circuit for restoring the digital signal is proposed. Finally, the transmission delay of signals is discussed.

II. CHUA'S CIRCUIT

Let us consider a chain interconnection of "n" identical Chua's circuits (cells). The state equation for Chua's circuit in Fig. 1 is given by

$$\begin{aligned} C_1\frac{dx}{dt} &= \frac{y-x}{R} - g(x), \\ C_2\frac{dy}{dt} &= \frac{x-y}{R} + z, \\ L\frac{dz}{dt} &= -y - rz, \end{aligned} \tag{1}$$

where x, y, and z are the voltage across C_1, the voltage across C_2, and the current through L, respectively. Here, a resistance r is added to the ideal Chua's circuit in order to account for the small parasitic resistance. The *v-i* characteristic of the non-

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linear resistor is given by

$$g(x) = G_b x + \frac{1}{2}(G_a - G_b)(|x + B_p| - |x - B_p|) + \frac{1}{2}(G_c - G_b)(|x + B_q| - |x - B_q|). \quad (2)$$

In our experiments, we used the following parameters:

$$C_1 = 10.17\,nF,\ C_2 = 103.0\,nF,\ L = 21.0\,mH,$$
$$R = 1480\,\Omega,\ r = 62.0\,\Omega,$$
$$G_a = -0.865\,mS,\ G_b = -0.525\,mS,$$
$$G_c = 2.30\,mS,\ B_p = 1.75\,V,\ B_q = 9.6\,V.$$

III. TRANSMISSION OF ANALOG SIGNALS

Consider a chain of "n" identical Chua's circuits coupled by identical linear resistors R_x, as shown in Fig. 2. An analog signal is applied to the chain by a current source $s(t)$. The output signal consists of the current through the i-th cell. In our experiments, seven cells are interconnected in this chain (that is, $n = 7$).

All cells are found to be *nearly synchronized* for the parameter value R_x ranging from 0 to 300 Ω. Figure 3 shows the experimental results for $R_x = 100\,\Omega$. Since an input signal $s(t)$ is applied to the chain and R_x is not zero, the cells are not completely synchronized, and therefore the current through the cells is corrupted by chaotic noises. We can reduce this chaotic noise by using a simple low-pass filter, since the frequency components of the noise is higher than those of the input signal. Observe the noise reduction effects shown in Fig. 4.

IV. TRANSMISSION OF DIGITAL SIGNALS

As the coupling resistance R_x increases, the following behavior is observed in the chain:
(a) noise increases and propagates through the cells, and the transmission quality becomes poorer.
(b) the cells in the chain are no longer nearly synchronized but are in disorder.

Therefore, a new method to transmit signals is required. The main idea of our proposed system is based on the following experimental phenomenon:
"Apply the sinusoidal signal $s(t) = A\,sin(2\pi f t)$ to the chain and assume the amplitude A is sufficiently large compared to the maximum magnitude of the chaotic waveform when $s(t) = 0$ and the frequency f is sufficiently small compared to the mean oscillation frequency of the waveform when $s(t) = 0$. Then, frequency entrainment of chaos occurs in the chain of Chua's circuits in the sense that the voltage x_i (the voltage across the capacitor C_1 of i-th cell) "tracks" or synchronizes with the drive signal $s(t)$ over a small range of *frequency deviations* " Δf as we modulate the frequency from $f - \Delta f$ to $f + \Delta f$.

Figure 5 shows the response of a cell when a sinusoidal wave is applied to a chain. Here, the response x_1 "tracks" or synchronizes with the upward and downward swings of the applied signal over a small range of frequency deviations. Therfore, if a binary signal is used as an information signal and the chaotic noise can be removed from the output signal by some filtering device, then transmitting a signal through a disordered chain is possible.

Figure 6 shows the proposed signal recovering circuit, which consists of two zero-crossing comparators and one low-pass filter. This circuit functions as follows: The first zero-crossing comparator restores a binary signal from the signal x_7. The low-pass filter is used to remove the chaotic noise from the output of the first zero-crossing comparator. The second zero-crossing comparator restores a binary signal from the distorted signal (due to the response of the low-pass filter). Since the frequency components of the information signal are assumed to be sufficiently lower than those of the chaotic noise, the chaotic noise can be removed by using the low-pass filter shown in Fig. 6.

Figure 7 shows the experimental results for $R_x = 5.1\,k\Omega$. Note that in this case, the information signal is completely recovered. Figures 8 and 9, respectively, show the chaotic attractors and their orbits in the (x_i, x_j)-plane. Observe that although the cells are not synchronized to each other, a signal can be transmitted through the chain.

V. TRANSMISSION DELAY

In our experimental circuit, we observed a propagation delay in successive cells. The delay time of this chain is around $0.53\,ms$ ($R_x = 5.1\,k\Omega$). The same phenomenon was reported in [4]. We conjecture that this delay partially comes from the response of the low-pass filter and from the phase delay from each cell. However, this interesting behavior has not been fully investigated yet.

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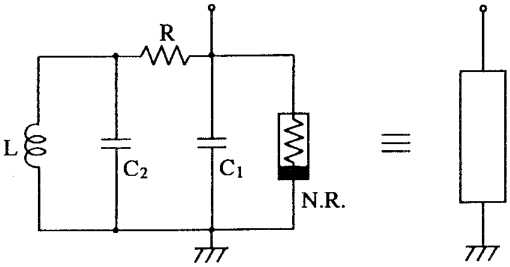

Fig. 1. Chua's circuit.

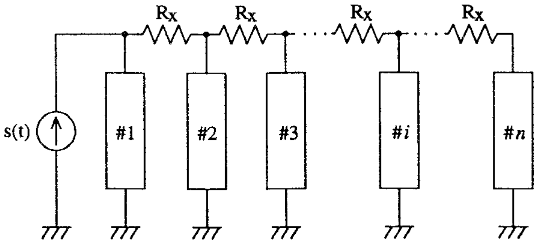

Fig. 2. A chain of Chua's circuits, coupled by resistance R_x.

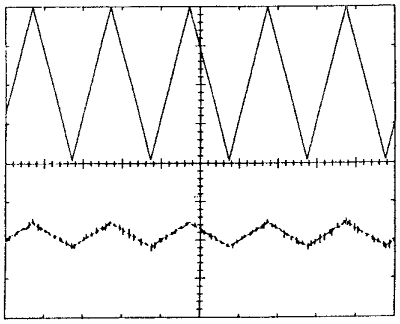

Fig. 3. Time waveforms of the current through the 7th cell (the bottom trace) and an input waveform (the top trace). The amplitude and the frequency of the input signal is $0.2mA$ and $500Hz$, respectively. Horizontal scale:1.0ms/div. Vertical scale:0.1mA/div. The oscilloscope figures are obtained by using a LEADER 3060D digital storage oscilloscope and a LEADER 715 plotter.

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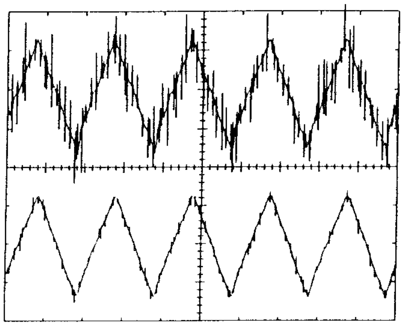

Fig. 4. Noise reduction effect. The top trace is the output signal corrupted by a chaotic noise. The bottom is the output signal which the chaotic noise is reduced by a low-pass filter. Horizontal scale:1.0ms/div. Vertical scale:0.1V/div.

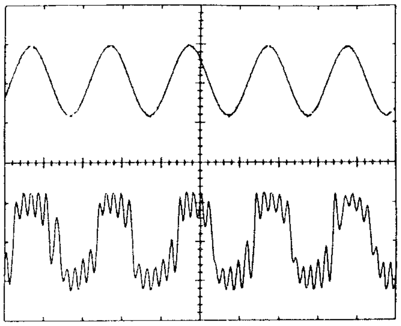

Fig. 5. Input vs response. The top trace is the sinusoidal input ($f = 250Hz$ and $A = 1.0mA$. Horizontal scale:2.0ms/div. Vertical scale:1.0mA/div.). The bottom trace is the response x_1 (Horizontal scale:2.0ms/div. Vertical scale:10V/div.).

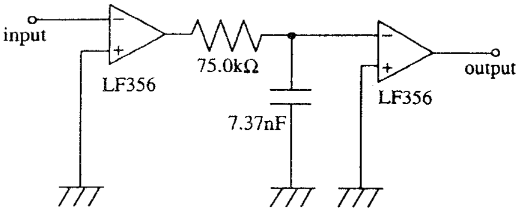

Fig. 6. Recovering system.

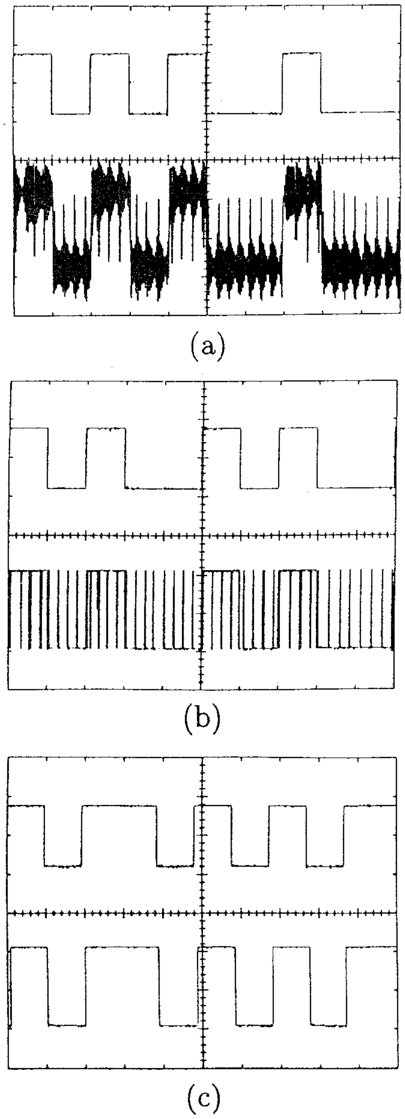

Fig. 7. Recovering process.
(a) The top trace is the input (Horizontal scale:10ms/div. Vertical scale:5.0mA/div.). The bottom trace is the response x_1 (Horizontal scale:10ms/div. Vertical scale:5.0/div.).
(b) The top trace is the input. The bottom trace is the recovered signal without the filter (Horizontal scale:10ms/div. Vertical scale:15V/div.).
(c) The top trace is the input. The bottom trace is the recovered signal with the filter (Horizontal scale:10ms/div. Vertical scale:15V/div.).

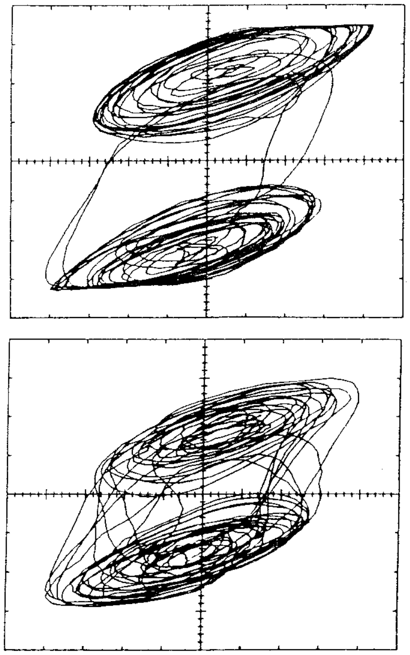

Fig. 8. Chaotic attractors of the 1st and the 7th cells. Horizontal scale:0.5V/div. Vertical scale:3.0V/div.

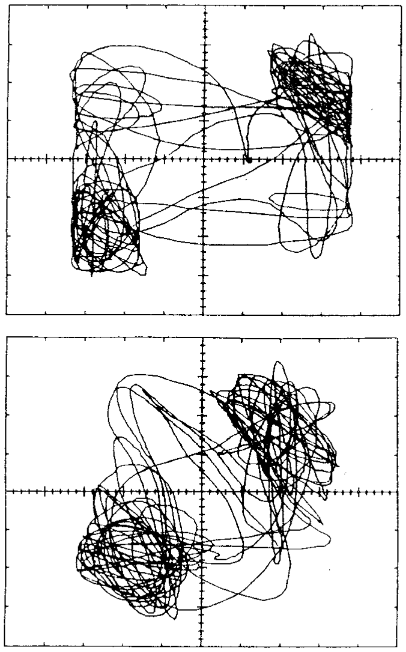

Fig. 9. Chaotic trajectories on the (x_1 , x_7) and (x_4 , x_7)-plane. Horizontal scale:3.0V/div. Vertical scale:3.0V/div.