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A new feedback control of a modified Chua's circuit system

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Abstract

A feedback controller is designed to suppress chaotic states of a modified Chua's circuit system. This controller is composed by the following two portions. One is the feedback part which constructs an equilibrium manifold by modifying the dynamics of the system. The other is the proportional feedback part which will control the system to desired states on the equilibrium manifold. The advantage of this method is that with this closed-loop controller the system attains faster settling time than the systems using previous controllers. Also, it will eliminate the tracking error without using other integral controllers.

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1. Introduction

Chaos appears frequently in nature and in man-made devices. To some extent, chaos is beneficial because it enhances reaction kinetics mechanism in transporting heat/mass transfer. On the other hand, chaos is undesirable because it causes irregular behaviour in nonlinear dynamical systems. Furthermore, chaotic behaviour is usually unpredictable in detail and may cause detrimental effects on some occasions. Therefore, the ability to control chaos (either promote or eliminate it) is practically important.

After the pioneering work on controlling chaos introduced by Ott, Grebogi and Yorke (OGY) [1], controlling chaos has become a fascinating topic in nonlinear dynamics. Generally speaking, there are two ways to control chaos: feedback control [1-3] and non-feedback control [4,5]. In this study, we focus on feedback control. Chua's circuit, suggested by L.O. Chua, has been studied extensively as a prototypical electronic system [11]. Chen and Dong [6,7] control the chaotic trajectory of the circuit to reach the limit cycle by using only linear feedback control. Using standard control methods, Hartley and Mossayebi [8,12] demonstrated the control of Chua's circuit systems. Hartley and Mossayebi also demonstrated how to design a controller for tracking the variable x of the system based on input-output and state-space techniques. The purpose of this paper is to present a new feedback control for Chua's circuit system in order to have a better performance than previous controllers.

A polynomial variant of the original Chua's circuit was presented by Hartley [9]. It was shown that the piecewise nonlinearity of Chua's circuit [10] could be

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replaced by a cubic nonlinearity with little change in the system dynamics or bifurcation structure. With this in mind, a control signal will be applied to the third state which would represent the addition of a voltage source in Chua's circuit. The modified Chua's circuit system is given by

$$\begin{cases} \dot{x} = p(y - f(x)) = p(y - \frac{1}{7}(2x^3 - x)) \\ \dot{y} = x - y + z \\ \dot{z} = -qy = -\frac{100}{7}y \end{cases}, \quad (1)$$

where $p > 0$ and $q > 0$ are system parameters; x and y are the voltages across two capacitors; and z is the current through the inductor. From the analysis of equilibrium point in system (1), the equilibrium points are $(\sqrt{0.5}, 0, -\sqrt{0.5})$, $(0,0,0)$, $(-\sqrt{0.5}, 0, \sqrt{0.5})$. When a voltage source u is added in series with the inductor, the dynamics of the controlled system is described by

$$\begin{cases} \dot{x} = p\left(y + \dfrac{x - 2x^3}{7}\right) \\ \dot{y} = x - y + z \\ \dot{z} = -\frac{100}{7}y + u \end{cases}. \quad (2)$$

In order to control the chaotic behaviour of Chua's circuit effectively, we propose a novel control concept.

The feedback controller u is expressed as follows:

$$\begin{aligned} u &= u^* + k_p(x_{\text{ref}} - x) \\ &= k(x - y + z) + qy + k_p(x_{\text{ref}} - x), \end{aligned} \quad (3)$$

where $u^* = k(x - y + z) + qy$ and $k_p(x_{\text{ref}} - x)$ is the proportional feedback part. The feedback part u^* is designed to modify the dynamics of the third equation of system (1), and leaves the second and third equations of system (2) unchanged. By adding u^*, the three equilibrium fixed points of system (1) will be extended as an equilibrium manifold, $y = f(x)$ and $z = y - x = f(x) - x$. The corresponding Jacobian matrix J and characteristic equation about this equilibrium manifold are, respectively,

$$J = \begin{bmatrix} p\left(-\dfrac{6x^2 - 1}{7}\right) & p & 0 \\ 1 & -1 & 1 \\ k & -k & k \end{bmatrix},$$

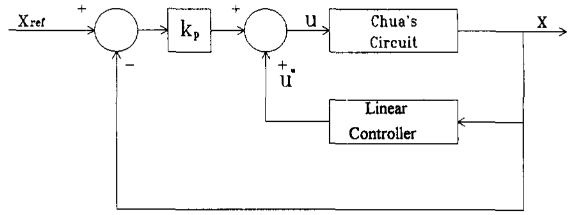


Fig. 1. Block diagram representation of the controlled system.

and

$$|\lambda I - J| = \lambda\Big(\lambda^2 + \Big(1 + p\Big(\frac{6x^2 - 1}{7}\Big) - k\Big)\lambda + p\Big(\frac{6x^2 - 1}{7}\Big)(1 - k) - p\Big) = 0. \quad (4)$$

The equilibrium manifold is linearly stable on two directions of eigenvectors when $(1 + p(\frac{6x^2-1}{7}) - k) > 0$ and $p(\frac{6x^2-1}{7})(1 - k) - p) > 0$. For tracking a desired state x_{ref}, we add the proportional feedback part $k_p(x_{\text{ref}} - x)$ to the controller u. The block diagram of the controlled system is shown in Fig. 1. The equilibrium manifold now becomes a new form, $x = x_{\text{ref}}$, $y = f(x_{\text{ref}})$, and $z = y - x = f(x_{\text{ref}}) - x_{\text{ref}}$. And the Jacobian and characteristic equation about this general equilibrium manifold will be respectively rederived as follows:

$$J = \begin{bmatrix} p\left(-\dfrac{6x_{\text{ref}}^2 - 1}{7}\right) & p & 0 \\ 1 & -1 & 1 \\ k & -k - k_p & k \end{bmatrix},$$

and

$$\begin{aligned} |\lambda I - J| &= \lambda^3 + \Big(1 - k + p\Big(\frac{6x_{\text{ref}}^2 - 1}{7}\Big)\Big)\lambda^2 \\ &\quad + \Big(k_p + p\Big(\frac{6x_{\text{ref}}^2 - 1}{7}\Big)(1 - k) - p\Big)\lambda \\ &\quad + p\Big(\frac{6x_{\text{ref}}^2 - 1}{7}\Big)k_p. \end{aligned} \quad (5)$$

Applying the Routh-Hurwitz criterion to Eq. (5), one can easily justify the stability criteria of the equilibrium manifold. In Fig. 2, we have plotted the stability results in the $x - k$ plane under different k_p values. It is found that by increasing k_p the stability region has been enlarged.

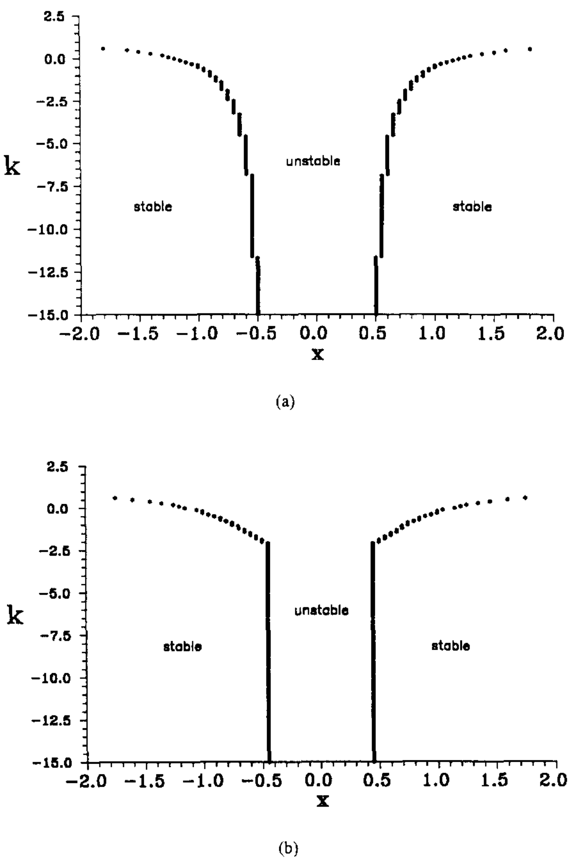


Fig. 2. The stability diagram in the $x - k$ plane. (a) $k_p = 1$. (b) $k_p = 10$.

In the following section, we will make a series of numerical experiments to verify the performance of our new nonlinear feedback controller. (The fourth-order Runge-Kutta was used with step size 0.02). In the numerical experiments, the parameters are set at $p = 10$, $q = 100/7$, and the initial condition is (0.65, 0, 0). Fig. 3 shows that under this set of conditions, the system will exhibit chaotic behaviour if no control is applied. The purpose of applying control to this system is to suppress its undesired chaotic states.

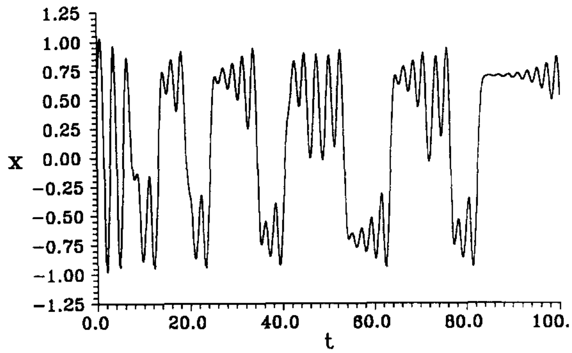


Fig. 3. The chaotic behaviour of the uncontrolled Chua's circuit system with the parameters $p = 10$, $q = 100/7$ and initial condition is (0.65, 0, 0).

2. Numerical experiments

2.1. Control the state to the original equilibrium point

Firstly, we let the reference input $x_{ref} = \sqrt{0.5}$ as the fixed point of original system (1). The control is activated at time $t = 40$. Fig. 4 shows that in such condition, if only using the P controller ($u = k_p(x_{ref} - x)$), the system will oscillate around the desired fixed point and finally lead to period errors. For solving the problem of tracking errors, Hartley and Mossayebi [8] adopted the state-space technique. Their controller is defined as $u = -Kx = -[k_x\ k_y\ k_z\ k_w][x\ y\ z\ w]^T$, where $w = \int(x_{ref} - x)dt$ and the optimal state feedback gain K is given by $[1.61\ \ 0.92\ \ 1.68\ \ -5.0]^T$. It is a useful method for eliminating steady state error. However, the system using state-space technique controller [8] will exhibit a larger overshoot value and a longer settling time than that using the present controller. Fig. 4 shows that our controller can successfully control the chaotic states and have a better performance.

2.2. Control the state to the new fixed point on the equilibrium manifold

Second, we let the reference input $x_{ref} = 0.5$ as the new fixed point. The control is also activated at time $t = 40$. Because the P controller will always lead to periodic errors, we do not display its result

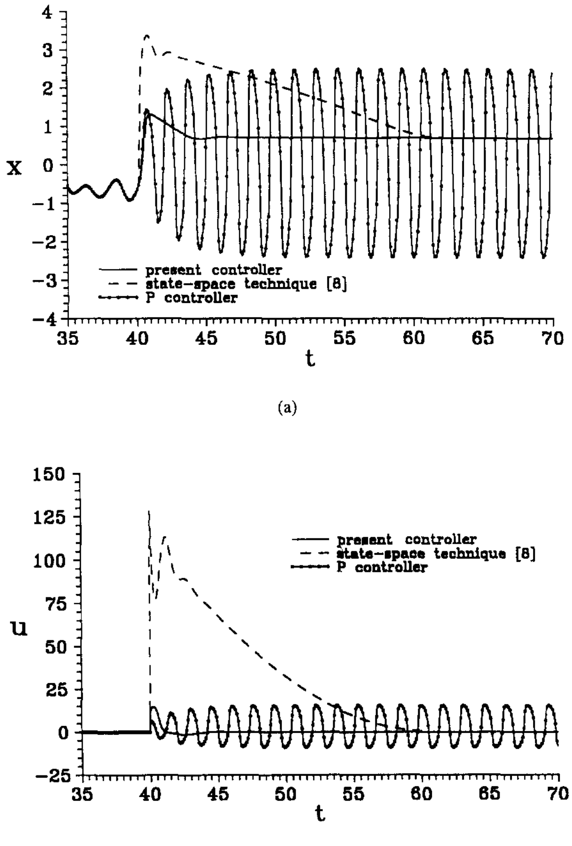


Fig. 4. Time responses of (a) the x-output for three different controllers and (b) the control input, u is activated at time $t = 40$, for the case of control the state to the original equilibrium point.

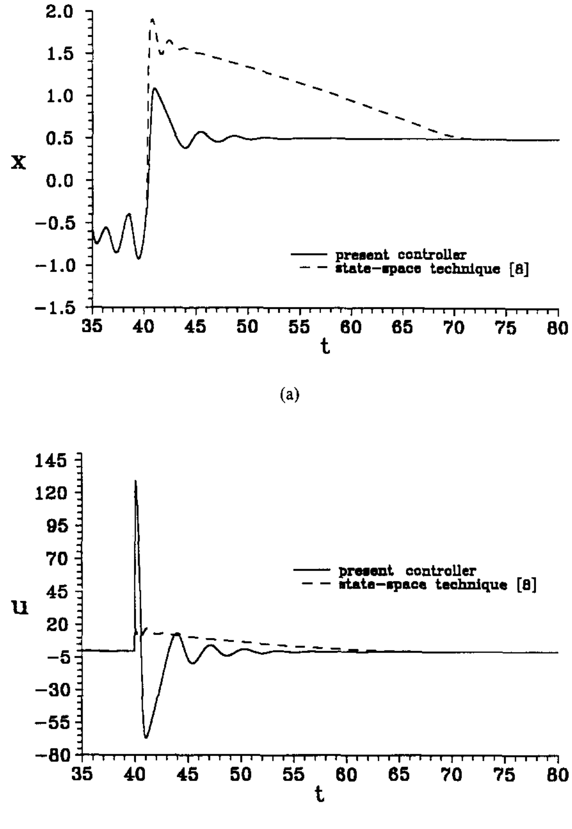
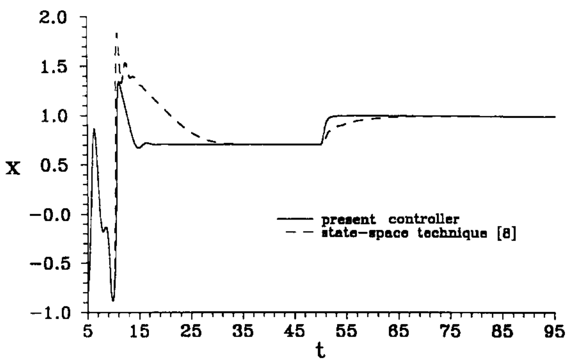


Fig. 5. Time responses of (a) the x-output for two different controllers and (b) the control input, u is activated at time $t = 40$, for the case of control the state to the new fixed point on the equilibrium manifold.

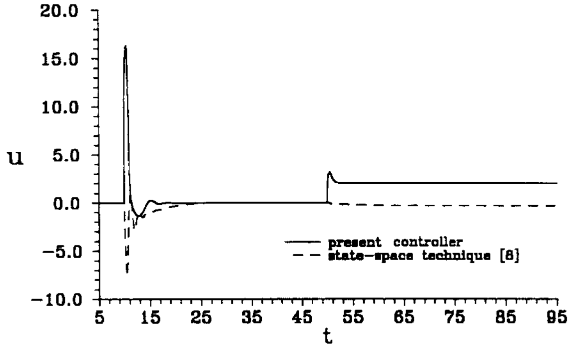
in the following. Fig. 5 shows that in this situation the qualitative results are similar to that of controlling the state to original equilibrium point. But here the system with both kind of controllers will exhibit a longer settling time than the previous case.

From the above numerical experiment, it is found that the system with the present nonlinear controller achieves faster settling time than that with the other two types of controllers. Now we want to study more general control process, it will steer the desired state from the fixed point $\sqrt{0.5}$ to another one. The control is activated at time $t = 10.0$ first and then x_{ref} is shifted from $\sqrt{0.5}$ to 1 at time $t = 50.0$. Fig. 6 displays the results of using state-space technique [8] and the present nonlinear controller. At $t = 10$, the system using the state-space technique [8] has a longer settling time than that using the present nonlinear controller. When we extend our desired state to $x_{\text{ref}} = 1$, the numerical result shows the system using the present controller also achieves faster settling time than that using state-space technique [8]. Therefore, this system using the present controller will give good performance in tracking the states along the general equilibrium manifold.

In Table 1, we summarize the quantitative results of three types controllers. For controlling the state to the original equilibrium point or a new fixed point on the equilibrium manifold, the present nonlinear controller has a faster settling time than that using state-space technique [8]. But if only using P controller the system always exhibits periodic errors.



(a)



(b)

Fig. 6. Time responses of (a) the x-output for two different controllers and (b) the control input, u is activated at time $t = 10$, for the case of control input is changed form $\sqrt{0.5}$ to 1 at time $t = 50$.

Table 1
Comparison of the settling times for different control schemes

Controller schemes	Settling time	
	$x_{ref} = \sqrt{0.5}$	$x_{ref} = 1.0$
controller proposed in this paper	3.97	1.24
controller proposed by Hartley and Mossayebi [8]	11.23	3.14
P controller	periodic error	periodic error

For solving the problem of tracking error, Hartley and Mossayebi [8] design a controller by using the state-space technique. For the state-space controller, they used optimal control theory to attain a set of optimal state feedback gains K. For comparison, we

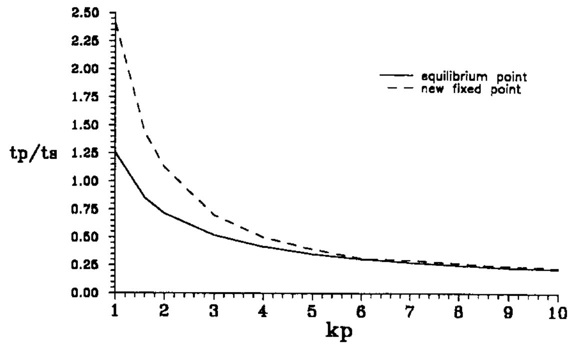
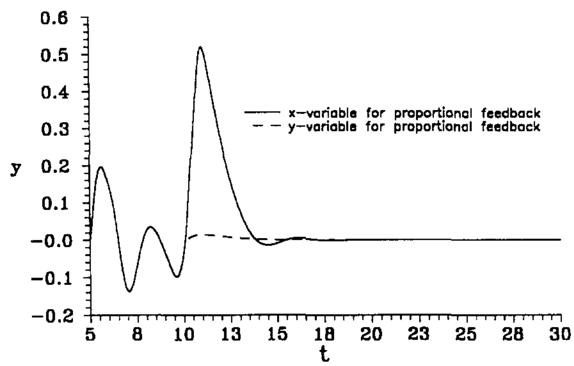


Fig. 7. Time responses of the y-output (a) for the case of control the state to the original equilibrium point. ($y_{ref} = 0$). (b) for the case of control the state to the new fixed point on the equilibrium manifold. ($y_{ref} = 1/7$).

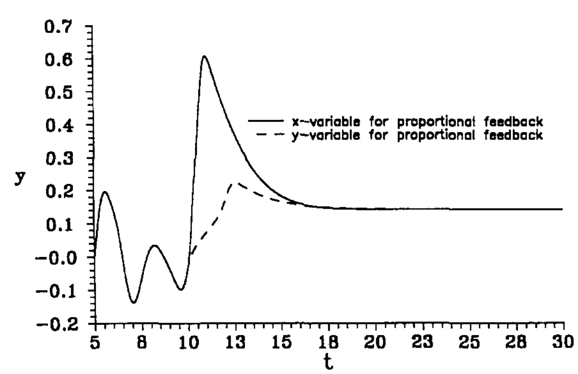
have displayed t_p/t_s vs. k_p in Fig. 7 (where t_p is the settling time of present nonlinear controller and t_s is the minimum settling time of state-space technique [8] with the optimal state feedback gain K). From Fig. 7, when k_p value of proportional feedback part increases, the settling time of control system will be shorter. Furthermore when $k_p > 1.5$ ($k_p > 2.3$) in the case of controlling the state to the original equilibrium point (a new fixed point on the equilibrium manifold), the present system will exhibit a faster settling time (i.e. $t_p/t_s < 1$) than that using state-space technique. For efficiency and reality, we can choose the value of k_p in a wider region. In the previous numerical experiments, we have used $k_p = 5$.

2.3. *y-variable is used as a reference state for proportional feedback*

As to Chua's circuit system, we also consider the case of using the y variable as a reference state for proportional feedback. In this case for the results of the Jacobian matrix, characteristic equation and the Routh-Hurwitz stability criterion we refer the reader to former work and we do not show it here. Fig. 8a shows that the control system with the y-variable as proportional feedback has a faster settling time than that with x-variable as a reference state in the case of controlling the state to the original equilibrium point ($y_{ref} = 0$). While Fig. 8b shows the case of controlling the state to the general fixed point on the equilibrium



(a)



(b)

Fig. 8. t_p/t_s vs. k_p for $k = -10$; t_p : the settling time of present nonlinear controller. t_s : the minimum settling time of state-space technique [8] with the optimal state feedback gain K.

manifold, both systems will have almost equivalent settling time.

3. Conclusion

We have developed a new strategy to control the chaotic states of a modified Chua's circuit system. This control method via feedback will eliminate partial dynamics of the system such that the equilibrium states will be extended to the equilibrium manifold. This procedure is a new concept in control theory. After using the Routh-Hurwitz criterion to determine its stability region, we investigate the case of controlling the states to the original equilibrium point of the system and the case of controlling the states to a new fixed point on the general equilibrium manifold respectively. It is found that:

(i) The present nonlinear controller may have a shorter settling time and lower overshoot than using previous controllers.
(ii) The present nonlinear controller can eliminate the tracking errors without using another integral controller.
(iii) When the gain of proportional feedback part increases, the settling time on the response of control system will be shorter.
(iv) When k_p is bigger than some value, the present system has a faster settling time (i.e. $t_p/t_s < 1$) than that using state-space technique.
(v) The control system with y-variable as a reference state for proportional feedback has a faster settling time than that with x-variable as a reference state in the case of controlling the state to the original equilibrium point. In the case of controlling the state to the general fixed point on the equilibrium manifold, both systems will have almost equivalent settling time.

The analysis of global and robust behaviour of this nonlinear control method remains for further research.

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