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Approximate Model-Matching for Nonlinear Systems with a Small Time-Delay; Application to a Chaotic Chua Circuit

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Abstract

Controller design procedures for nonlinear systems with small time-delay in the control input are proposed. These controllers solve approximately the model-matching problem. The delayed nonlinear system is represented in an approximate form by a system having a singular perturbation structure free from time-delays. The controller is an improved version of that one corresponding to a design where the delay is neglected. An application to a Chua system presenting a short delay in the control input, and displaying regular and chaotic dynamics, illustrates the proposed procedure.

1 Introduction

One of the most exciting and interesting ideas developed in the past few decades is the complex and chaotic behavior of dynamical systems. At present, practical implications of these ideas are leading to novel applications where chaos must be controlled. Among the kind of chaotic systems, the Chua circuit has an important place. Different conventional control techniques have been proposed to make this system display a regular or a chaotic behavior [8]. Because this is a nonlinear system, the use of nonlinear control techniques may yield better results when the control objectives are more complicated than merely to regulate an equilibrium point; for example, tracking of periodic orbits or matching the behavior of a given model. In this sense, the matching of the dynamical behavior of the plant to that given by a reference model, problem known as model-matching, can give excellent results when the plant and the model has similar behavior. In the case of a chaotic plant matched to a chaotic model, the versatile dynamics of the reference model may produce a closed-loop system that can display, in an efficient way, a great variety of dynamical behaviors: equilibrium points, periodic orbits of different periods, or chaotic trajectories.

On the other hand, systems with time-delays are found in many situations, for instance, mechanical components, electronic circuits, teleoperators, etc. Some procedures to control these systems involve input-output decoupling and disturbance decoupling techniques [7], [10]. In many cases, the feedback laws result non causal. Time-delays appear also in electronic circuits, including the Chua oscillator [2], introducing phase-lags that are very difficult to cope with.

In this paper we propose a procedure to design controllers for nonlinear systems with small time-delays in the control input. These controllers solve approximately the model-matching problem. The delayed nonlinear system is represented in an approximate form by a system having a singular perturbation structure free from time-delays. Thus, by using singular perturbation theory [6], together with some results from the geometric nonlinear control literature [4], a model-matching controller can be designed and the stability analysis can be carried out. The approach presented in this work is based on similar ones followed independently by Sannuti [9] and Inuoe et al [3] for linear systems. We illustrate the procedure with an application to a controlled Chua system having a control input with a short time-delay. In this application the plant is fixed, and the reference model is another Chua system that can display a variety of attractors by varying slightly one parameter, ranging from equilibrium points to chaotic attractors.

2 Problem description

Let us consider the nonlinear time-delay system

$$\begin{aligned} \dot{\xi}^P(t) &= f^P\left(\xi^P(t)\right) + g^P\left(\xi^P(t)\right) u(t-\tau), \quad (1) \\ y^P(t) &= h^P\left(\xi^P(t)\right), \end{aligned}$$

where $\xi^P \in \Xi^P \subset \mathbb{R}^n$, Ξ^P is an open set containing the origin; u, $y^P \in \mathbb{R}$, $f^P(0) = 0$, $h^P(0) = 0$. Consider

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also a nonlinear model given by

$$\begin{aligned}\dot{\xi}^M(t) &= f^M(\xi^M(t)) + g^M(\xi(t))\, u^M(t), \quad (2)\\ y^M(t) &= h^M(\xi^M(t)),\end{aligned}$$

where $\xi^M \in \Xi^M \subset \mathbb{R}^n$, Ξ^M is an open set containing the origin; u^M, $y^M \in \mathbb{R}$, $f^M(0) = 0$, $h^M(0) = 0$. We define the model-matching problem.

Definition 1 (Model-matching problem) *Consider the system (1) and the model (2). Find, if possible, a regular static state feedback*

$$u(t) = \gamma(\xi^P(t), \xi^M(t), u^M(t)), \qquad (3)$$

defined in an open subset of $\Xi^P \times \Xi^M \times \mathbb{R}$, *such that there exists a domain* $D \subset \Xi^P \times \Xi^M$ *containing the origin and, if* $(\xi^P(0), \xi^M(0)) \in D$, *then the closed-loop system yields* $\lim_{t\to\infty}(y^M(t) - y^P(t)) = 0$.

Suppose that the relative degrees about the origin of system (1) with respect to $u(t-\tau)$, and of system (2) with respect to $u^M(t)$, are both n. It is known [4] that it is possible to find local coordinates transformations

$$x^P = T^P(\xi^P) = \left(h^P, L_{f^P}h^P, \ldots, L_{f^P}^{n-1}h^P\right)(\xi^P)$$

and

$$x^M = T^M(\xi^M) = \left(h^M, L_{f^M}h^M, \ldots, L_{f^M}^{n-1}h^M\right)(\xi^M)$$

such that (1) and (2) will be given by

$$\begin{aligned}\dot{x}_i^P(t) &= x_{i+1}^P(t), \ (i = 1, \ldots, n-1),\\ \dot{x}_n^P(t) &= f_n^P(x^P(t)) + g_n^P(x^P(t))\, u(t-\tau),\\ y^P(t) &= x_1^P(t),\end{aligned}$$

and

$$\begin{aligned}\dot{x}_i^M(t) &= x_{i+1}^M(t), \ (i = 1, \ldots, n-1),\\ \dot{x}_n^M(t) &= f_n^M(x^M(t)) + g_n^M(x^M(t))\, u^M(t),\\ y^M(t) &= x_1^M(t),\end{aligned}$$

where

$$f_n^P = L_{f^P}^n h^P(\xi^P), \quad g_n^P = L_{g^P}L_{f^P}^{n-1}h^P(\xi^P) \neq 0,$$

$$f_n^M = L_{f^M}^n h^M(\xi^M), \quad g_n^M = L_{g^M}L_{f^M}^{n-1}h^M(\xi^M) \neq 0,$$

$$\xi^P = (T^P)^{-1}(x^P), \quad \xi^M = (T^M)^{-1}(x^M),$$

$x^P \in X^P = T^P(\Xi^P)$, and $x^M \in X^M = T^M(\Xi^M)$. Let us suppose that the model is a self-excited system with bounded trajectories, so $u^M \equiv 0$. Define $x = (x^M, x^P)$, $y = y^M - y^P$. Therefore, we can define a global system having the form

$$\begin{aligned}\dot{x}(t) &= f(x(t)) + g(x(t))\, u(t-\tau), \quad (4)\\ y(t) &= h(x(t)),\end{aligned}$$

where $f = (x_2^M, \ldots, x_n^M, f_n^M, x_2^P, \ldots, x_n^P, f_n^P)$, $g = (0, \ldots, 0, g_n^P)$, $h = x_1^M - x_1^P$. The model-matching problem is then reduced to the stabilization of the origin of this system.

If we follow the same technique used to determine the control law for systems without delay ($\tau = 0$) to calculate the control law that solves this problem for systems with delay ($\tau > 0$), then we obtain the control

$$u(t) = \frac{f_n^M(x^M(t+\tau)) - f_n^P(x^P(t+\tau)) - r(t+\tau)}{g_n^P(x^P(t+\tau))}, \qquad (5)$$

where r is a new input. The noncausality of this control law makes it inapplicable; therefore, we will consider an approximation of (4) that is free from delays and permits to obtain a causal control law that solves approximately the model-matching problem. We also discuss the stability of the closed-loop system.

3 Approximate model-matching control

Let us denote the delay by $\tau = k\varepsilon$, where ε is a small, positive number, and k is a positive integer. Now define $z_i(t) = u(t - i\varepsilon)$, $i = 1, \ldots, k$, and consider the approximation $\dot{z}_i(t) \cong (z_i(t+\varepsilon) - z_i(t))/\varepsilon$. Therefore, we obtain

$$\varepsilon\dot{z}(t) = Az(t) + bu(t), \qquad (6)$$

where $z = (z_1, \ldots, z_k)$, $b = (1, 0, \ldots, 0)^T \in \mathbb{R}^k$, and A is the $(k \times k)$-matrix

$$A = \begin{pmatrix} -1 & 0 & & \cdots & 0\\ 1 & -1 & 0 & \cdots & 0\\ \vdots & & \ddots & & \vdots\\ 0 & \cdots & 1 & -1 & 0\\ 0 & \cdots & 0 & 1 & -1 \end{pmatrix}.$$

Using (6), the nonlinear system (4) can be approximated by

$$\begin{aligned}\dot{x} &= \tilde{f}(x, v) \stackrel{\text{def}}{=} f(x) + g(x)v,\\ \varepsilon\dot{z} &= Az + bu,\\ v &= cz, \qquad (7)\\ y &= h(x),\end{aligned}$$

where c is the k-row vector $(0, \ldots, 0, 1)$, and all the signals are evaluated at the present time t. Note that $\tilde{f}(0,0) = 0$ and the matrix A is strictly Hurwitz, with all its eigenvalues at -1. Now we want to find the conditions such that the output y converges to zero, remaining bounded the state trajectories of (7).

Assume that, for a given input u, the solution of the z-subsystem of (7) is given, for ε small enough, by

$$z(t) = \phi(t, \varepsilon) = \sum_{i=0}^{\infty} \varepsilon^i \phi_i(t). \qquad (8)$$

In the same way, assume that the control law u can be expressed as

$$u = \sum_{i=0}^{\infty} \varepsilon^i u_i. \tag{9}$$

Using (8-9) into (6) leads to

$$\sum_{i=0}^{\infty} \varepsilon^{i+1} \dot{\phi}_i = \sum_{i=0}^{\infty} \varepsilon^i \left(A\phi_i + bu_i \right).$$

By grouping the corresponding coefficients of ε^i we obtain

$$A\phi_0 + bu_0 = 0, \quad \dot{\phi}_i = A\phi_{i+1} + bu_{i+1}, \ i \geq 0,$$

from where it is possible to get

$$\phi_i = -\sum_{j=0}^{i} A^{-(i+1-j)} b \frac{d^{i-j} u_j}{dt^{i-j}}, \ i \geq 0. \tag{10}$$

Now using (8) and (10) into the x-subsystem of (7) we find

$$\dot{x} = f(x) + g(x)\tilde{u} - g(x) c \sum_{i=1}^{\infty} \varepsilon^i \sum_{j=0}^{i} A^{-(i+1-j)} b \frac{d^{i-j} u_j}{dt^{i-j}}, \tag{11}$$

where $\tilde{u} = -cA^{-1}bu_0 = u_0$ because $cA^{-1}b = -1$. Therefore, if we set

$$c \sum_{i=1}^{\infty} \varepsilon^i \sum_{j=0}^{i} A^{-(i+1-j)} b \frac{d^{i-j} u_j}{dt^{i-j}} = 0, \tag{12}$$

then (11) reduces to $\dot{x} = f(x) + g(x)\tilde{u}$. Therefore, we propose to design $\tilde{u}$ to stabilize this last system, with condition (12) fulfilled. From a practical point of view we can truncate the series on ε up to a finite order, obtaining an approximate solution. For instance, if we consider only the first-order approximation, then from (10) and (12) we will have $\phi_0 = -A^{-1}bu_0$, $\phi_1 = -A^{-2}b\dot{u}_0 - A^{-1}bu_1$, and $u_1 = cA^{-2}b\dot{u}_0 = k\dot{u}_0$ because $cA^{-2}b = k$.

The reduced-order model, obtained with $\varepsilon = 0$, will be

$$\begin{aligned} \dot{x} &= \tilde{f}(x, u_0) = f(x) + g(x) u_0, \\ y &= h(x). \end{aligned} \tag{13}$$

Then the state feedback control law

$$u = u_0(x, r) \stackrel{\text{def}}{=} \frac{f_n^M\left(x^M(t)\right) - f_n^P\left(x^P(t)\right) - r(t)}{g_n^P\left(x^P(t)\right)}, \tag{14}$$

with $r(t) = -\boldsymbol{\alpha} \cdot \mathbf{y}$, $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)$, $\mathbf{y} = (y_1, \ldots, y_n)$, $y_i = x_i^M - x_i^P$, and α_i such that $\lambda^n + \alpha_n \lambda^{n-1} + \cdots + \alpha_1$ is strictly Hurwitz, yields $\lim_{t\to\infty} y(t) = 0$, keeping bounded the state trajectories of this system. In fact, the reduced system (13) with the control law (14) has the form

$$\begin{aligned} \dot{y}_i(t) &= y_{i+1}(t), \ (i = 1, \ldots, n-1), \\ \dot{y}_n(t) &= -\boldsymbol{\alpha} \cdot \mathbf{y}(t), \\ \dot{x}_i^M(t) &= x_{i+1}^M(t), \ (i = 1, \ldots, n-1), \\ \dot{x}_n^M(t) &= f_n^M\left(x^M(t)\right), \\ y(t) &= y_1(t). \end{aligned} \tag{15}$$

It is evident that, if the origin of the model is stable, then this reduced system will also have the origin as a stable equilibrium point.

Remark When $\varepsilon \neq 0$, the first-order approximate control law is given by $u = u_0 + \tau\dot{u}_0$, which corresponds to the first-order approximation of (5) with respect to τ.

In the $\left(\mathbf{y}, x^M, z\right) \equiv \left(x^G, z\right)$ –coordinates, the complete closed-loop system is given by

$$\begin{aligned} \dot{x}^G(t) &= f^G\left(x^G(t), z(t), \varepsilon\right), \\ \varepsilon\dot{z}(t) &= g^G\left(x^G(t), z(t), \varepsilon\right), \\ y(t) &= y_1(t), \end{aligned}$$

where

$$f^G = \left(y_2, \ldots, y_{n-1}, f_n^G, x_2^M, \ldots, x_{n-1}^M, f_n^M\right),$$
$$f_n^G = f_n^M\left(x^M\right) - f_n^P\left(x^M - \mathbf{y}\right) - g_n^P\left(x^M - \mathbf{y}\right) cz,$$
$$g^G = Az + bu\left(x^G\right),$$
$$u = u_0\left(x^G\right) + k\varepsilon\,\dot{u}_0\left(x^G\right) + \cdots,$$

and u_0 given by (14) with $x^P = x^M - \mathbf{y}$. We then have a singularly perturbed system where the reduced, "nominal" closed-loop system (15) has the origin as an equilibrium point whose stability depends on the model. On the other hand, the boundary-layer model $\dot{\eta} = A\eta$, where $\eta = z + A^{-1}bu_0$, is independent of x^G, linear, and exponentially stable. Furthermore, note that

1. $f^G(0,0,\varepsilon) = 0$, $g^G(0,0,\varepsilon) = 0$.
2. $g^G\left(x^G, z, 0\right) = Az + bu_0\left(x^G\right) = 0$ has an isolated root $z = h\left(x^G\right) = -A^{-1}bu_0\left(x^G\right)$ such that $h(0) = 0$.
3. The origin of the reduced system $\dot{x}^G(t) = f^G\left(x^G(t), h\left(x^G(t)\right), 0\right)$ is exponentially stable if the origin of the model does it.

Then we can conclude exponential stability of the origin of the complete system (7) for a small enough delay $\tau = k\varepsilon$ [5].

Remark Exponential stability of the origin is accomplished if the model has the origin as an exponentially stable equilibrium point. This is a sufficient condition for stability if ε is small enough. In the case where the model has bounded trajectories only, it is also possible to have convergence of the error $y^M - y^P$ to zero and boundedness of the state trajectories. This is illustrated with the example discussed below.

Remark The proposed control law $u = u_0 + k\varepsilon\, du_0/dt + \cdots + c(\varepsilon^m)\, d^m u_0/dt^m$ is known as an mth-order approximate solution for the model-matching problem defined before, applied to the system in singular perturbation form (7) that approximates the original delayed nonlinear sytstem (1). The zero-order approximation can be obtained by setting $\varepsilon = 0$ (so $u = u_0$), which is equivalent to design a control law for the delayed system by neglecting the time-lag.

4 Application to the Chua system

In this section we apply the proposed control law to a delayed Chua system. A schematic diagram of the Chua circuit is shown in figure 1 [1]. It is described by

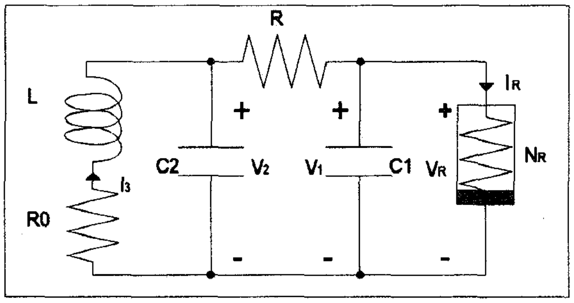

Figure 1: Schematic diagram of the Chua system.

$$\begin{aligned} C_1\dot{v}_1 &= G(v_2 - v_1) - f_{NL}(v_1), \\ C_2\dot{v}_2 &= G(v_1 - v_2) + i_3, \\ L\dot{i}_3 &= -(v_2 + R_0 i_3), \end{aligned} \tag{16}$$

where C_1, C_2, L, $G = 1/R$, and R_0 are real numbers, and

$$f_{NL}(v_R) = G_b v_R + \frac{1}{2}(G_a - G_b)(|v_R + E| - |v_R - E|) \tag{17}$$

is the $v - i$ characteristics of the nonlinear resistance N_R, which is a piece-wise linear function with slopes G_a and G_b and corners at $v_R = \pm E$. A change of variables given by

$$\begin{aligned} x_1 &= \frac{v_1}{E}, \quad x_2 = \frac{v_2}{E}, \quad x_3 = \frac{i_3}{GE}, \\ \alpha &= \frac{C_2}{C_1}, \quad \beta = \frac{C_2}{LG^2}, \quad \mu = \frac{R_0 C_2}{LG}, \\ a &= \frac{G_a}{G}, \quad b = \frac{G_b}{G}, \quad t \to \frac{t}{RC_2}, \end{aligned}$$

leads to the adimensional model

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1 - f_{NL}(x_1)), \\ \dot{x}_2 &= x_1 - x_2 + x_3, \\ \dot{x}_3 &= -\beta x_2 - \mu x_3, \end{aligned} \tag{18}$$

with

$$f_{NL}(x_1) = bx_1 + \frac{1}{2}(a - b)(|x_1 + 1| - |x_1 - 1|). \tag{19}$$

This system can display a variety of limit sets. Figure 2 shows some responses obtained from the system, from equilibrium points to strange attractors, for the parameter values $\beta = 16.5811$, $\mu = 0.13083$, $a = -1.39386$, $b = -0.75590$, and different values of α.

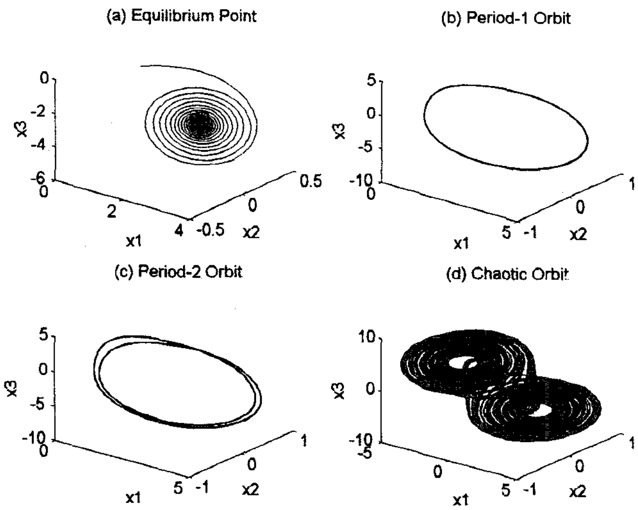

Figure 2: Trajectories of the Chua system. (a) $\alpha = 7.7189$, (b) $\alpha = 9.0066$, (c) $\alpha = 9.100$, (d) $\alpha = 10.0063$.

A controlled Chua system with time-delay in the input can be described by

$$\begin{aligned} \dot{\xi}_1^P(t) &= \alpha^P\left[\xi_2^P(t) - \xi_1^P(t) - f_{NL}^P(\xi_1^P(t))\right] \\ &\quad + u(t - \tau), \\ \dot{\xi}_2^P(t) &= \xi_1^P(t) - \xi_2^P(t) + \xi_3^P(t), \\ \dot{\xi}_3^P(t) &= -\beta^P \xi_2^P(t) - \mu^P \xi_3^P(t), \\ y^P(t) &= \xi_3^P(t), \end{aligned} \tag{20}$$

with $f_{NL}^P(\cdot)$ given by (19). This corresponds to the injection of a control current in the node corresponding to capacitor 1 to control the inductance current. This system can be given the normal form

$$\begin{aligned}\dot{x}_i^P(t) &= x_{i+1}^P(t),\ (i=1,2),\\ \dot{x}_3^P(t) &= f_3^P\left(x^P(t)\right)+g_3^P\left(x^P(t)\right)u(t-\tau),\\ y^P(t) &= x_1^P(t),\end{aligned}$$

where

$$x_1^P=\xi_3^P,\ x_2^P=-\beta^P\xi_2^P-\mu^P x_3^P,\ g_3^P=-\beta^P,$$

$$x_3^P=-\beta^P\xi_1^P+\beta^P\left(\mu^P+1\right)\xi_2^P+\left(\left(\mu^P\right)^2-\beta^P\right)\xi_3^P,$$

$$\begin{aligned}f_3^P &= -\alpha^P\beta^P x_1^P-\left[\beta^P+\mu^P\left(1+\alpha^P\right)\right]x_2^P\\ &\quad -\left(\alpha^P+\mu^P+1\right)x_3^P+\alpha^P\beta^P f_{NL}^P\left(x^P\right),\\ f_{NL}^P &= \beta^P z^P+\frac{1}{2}\left(a^P-b^P\right)\left(\left|z^P+1\right|-\left|z^P-1\right|\right),\\ z^P &= -\frac{\beta^P+\mu^P}{\beta^P}x_1^P-\frac{\mu^P+1}{\beta^P}x_2^P-\frac{1}{\beta^P}x_3^P.\end{aligned}$$

Now consider a reference model with the same structure of the plant. The zero-order approximate control law (14) has the form

$$\begin{aligned}u_0 &= \frac{\alpha^M\beta^M x_1^M-\alpha^P\beta^P x_1^P}{\beta^P}\\ &\quad+\frac{\beta^M+\mu^M+\alpha^M\mu^M}{\beta^P}x_2^M\\ &\quad-\frac{\beta^P+\mu^P+\alpha^P\mu^P}{\beta^P}x_2^P\\ &\quad+\frac{\left(\alpha^M+\mu^M+1\right)x_3^M-\left(\alpha^P+\mu^P+1\right)x_3^P}{\beta^P}\\ &\quad-\frac{\alpha^M\beta^M f_{NL}^M\left(x^M\right)-\alpha^P\beta^P f_{NL}^P\left(x^P\right)}{\beta^P}+\frac{r}{\beta^P}.\end{aligned}$$

If we choose $r=-\alpha_1 y_1-\alpha_2 y_2-\alpha_3 y_3$, with $y_i=x_i^M-x_i^P$, and α_i such that $\lambda^3+\alpha_3\lambda^2+\alpha_2\lambda+\alpha_1$ be strictly Hurwitz, then we can obtain, for an small enough delay τ, an output error converging asymptotically to zero, with bounded state trajectories.

A first-order approximate control law is given by $u=u_0+k\varepsilon\dot{u}_0$. Note that, due to the time derivative present in this expression, the nonlinear functions f_{NL}^P and f_{NL}^M must be at least C^1. This is not the case for the Chua system (see equation (19)), so for the calculation of the control law we have approximated these functions to the form

$$\begin{aligned}f_{NL}(x) &\approx bx+\frac{a-b}{2}(x+1)\tanh\left[\lambda(x+1)\right]\\ &\quad-\frac{a-b}{2}(x-1)\tanh\left[\lambda(x-1)\right],\end{aligned}$$

with a large value of λ.

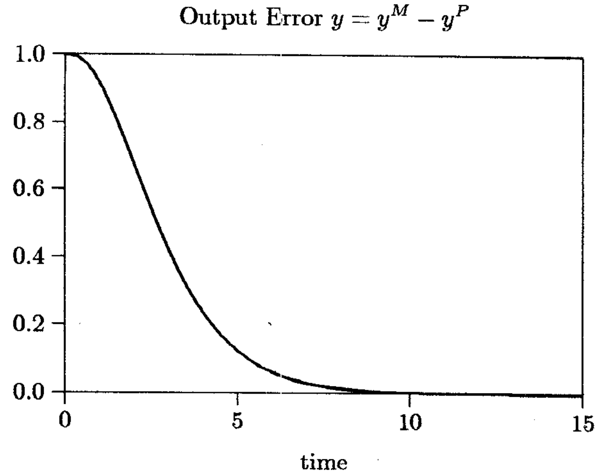

Figure 3: Typical output error of the controlled Chua system without delay, for a reference model displaying an equilibrium point, a periodic oscillation, or a chaotic attractor.

Note that the fast subsystem (6) has all its k poles placed at $-1/\varepsilon$. Therefore, to maintain a good separation between the "fast" dynamics induced by the delay, and the "slow" dynamics corresponding to the closed-loop reduced system (13), we have placed all the n poles of this last subsystem at $-1/p\varepsilon$, with p a large integer.

In figure 3 we show a typical output error $y=y^M-y^P$ obtained when the plant has an open-loop response corresponding to a stable equilibrium point (figure 2.a, $\alpha^P=7.7189$) with no time-delay ($\tau=0$), and the reference model is set to have a steady state given by an equilibrium point, a periodic oscillation, or a strange attractor. In all cases the output error is completely similar.

On the other hand, figure 4 shows the same output error when the plant has a stable equilibrium point ($\alpha^P=7.7189$), the reference model is displaying a chaotic behavior ($\alpha^M=10.0063$), for different time-delays. In this figure, the dotted lines correspond to the zero-order approximate control, the solid lines to the improved first-order approximate control law. Note the better response of this last controller.

Finally, figure 5 compares two attractors obtained from the model and those corresponding to the plant, for a time-delay $\tau=0.05$ and the improved control law: a strange attractor (figures 5.a and 5.b, $\alpha^M=10.1$), and a period-2 oscillation (figure 5.d). Figure 5.c shows the control signal for the chaotic case. Note the perfect reproduction of the attractors.

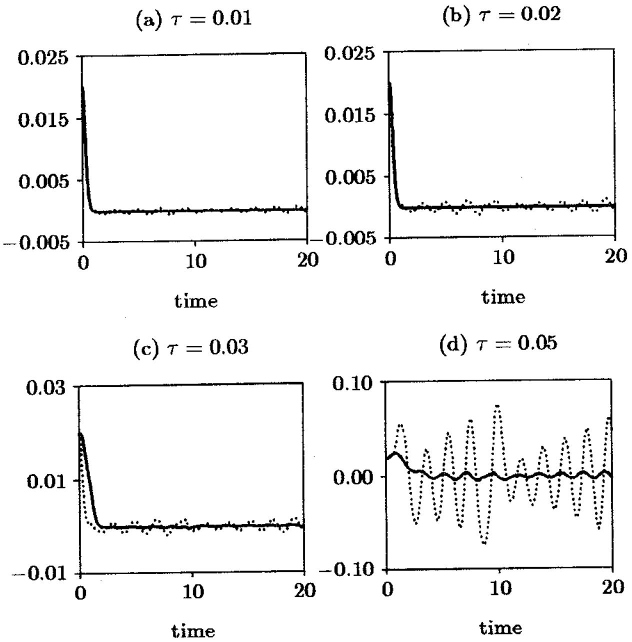

Figure 4: Output error $y = y^M - y^P$ of the controlled Chua system ($\alpha^P = 7.7189$) with different time-delays and a chaotic reference model ($\alpha^M = 10.0063$). Zero-order (dotted) and first-order (solid) approximation.

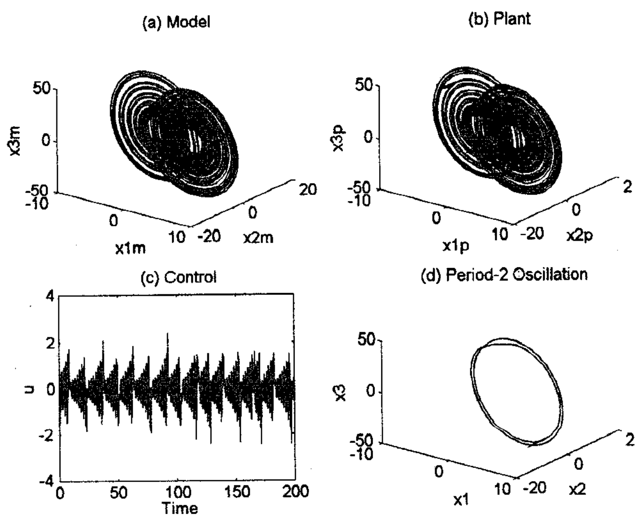

Figure 5: Model-matching of the Chua system with a time delay $\tau = 0.05$ and an improved first-order approximate control. (a) Model attractor. (b) Plant attractor. (c) Control signal for the chaotic attractor. (d) A period-2 oscillation: model (dashed) and plant (solid).

5 Conclusions

In this paper we have proposed a model-matching controller for a class of nonlinear systems with time-delay in the control input. The controller is an improved version of that one corresponding to a design where the delay is neglected. The stability analysis of the closed-loop system has been carried out by approximating the delayed system to a nonlinear system with a singular perturbation structure. The application of this control law has been illustrated with a Chua system presenting a short delay in the control input; this system can display regular and chaotic dynamics. This controller has made the delayed plant follow perfectly a variety of steady-state behaviors, ranging from equilibrium points, simple limit cycles, period-2 oscillations and chaotic trajectories.

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