Advanced Analog Integrated Circuits

Electronic Noise

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Types of Noise

- Interference, “Man-made” Noise
  - Substrate coupling
  - Supply noise
  - Signal coupling
  - Solutions:
    - Fully differential circuits
    - Layout techniques
    - Shielding

- Electronic noise
  - Fundamental physics based
    - Thermal and shot noise
  - Technology related
    - Flicker noise, drift
Noisy Signals
Thermal Noise Manifestations

- Affects all “dissipative” systems
  - E.g. Brownian motion

- Observable when signals are small*
  - Sensor interfaces (μV signals …)
  - RF receivers
  - Inertial sensors (e.g. Gyroscopes)

- Increasingly important as signals get smaller
  - Reduced supply voltage
  - Small sensors
  - High sensitivity radios

* Interestingly noise power is constant, “low noise design” is achieved by ensuring that signal power is larger than the noise power
Example: Resistor

- Carriers collide randomly with lattice atoms giving rise to small current variations over time
- Model as random noise current source $i_n(t)$ in parallel with ideal (noiseless) resistor
Properties of Thermal Noise
Thermal Noise Power Spectrum
Resistor Noise Models

**Voltage Noise Model**

\[ \overline{v_n^2} = P_n R = 4k_B T R \Delta f \]

**Current Noise Model**

\[ \overline{i_n^2} = \frac{P_n}{R} = \frac{4k_B T}{R} \Delta f \]

For \( R = 1\,\text{k}\Omega \) at RT:
Sloppy Nomenclature

• For convenience, noise in circuits is usually represented by the mean squared noise voltage $v_n^2$ or current $i_n^2$

• It is customary to refer to these quantities as “noise power”, especially when comparing them to signals which are usually represented by voltages or currents, not power

• The actual noise power is easily obtained by dividing or multiplying the mean squared values by the resistance
Resistors in Series

- Since $v_{n1}(t)$ and $v_{n1}(t)$ are uncorrelated

- **Beware**: mean squares add, not rms values!
Signal-to-Noise Ratio

Power delivered by source:

Signal-to-noise ratio:

Minimum power for given SNR:

Example:

Minimum power required is proportional to information processed.
Noise Bandwidth

- What is the noise bandwidth, $\Delta f$?
Band-Limited Noise Example

Noise PSD across resistor:

Total noise:

Total integrated noise is set by capacitance and independent of $R$. 
RC Noise Spectral Density
Useful Integrals

\[ \int_0^\infty \left| \frac{1}{1 + \frac{s}{\omega_o}} \right|^2 \, df = \frac{\omega_o}{4} \]

\[ \int_0^\infty \left| \frac{1}{1 + \frac{s}{\omega_oQ} + \frac{s^2}{\omega_o^2}} \right|^2 \, df = \frac{\omega_oQ}{4} \]

\[ \int_0^\infty \left| \frac{s}{\omega_o} \right|^2 \, df = \frac{\omega_oQ}{4} \]

Equipartition Theorem
Equivalent Noise Bandwidth

- Defined as the bandwidth of a brick-wall filter that results in the same total noise as the filter in question
- For a $1^{\text{st}}$ order RC filter, the equivalent bandwidth is $\frac{\pi}{2}$ times its 3-dB bandwidth
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Noise–Power Tradeoff

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Analog versus Digital SNR

- Bits versus SNR

- Signal-to-Quantization-Noise Ratio (see EE 240C)
  - Quantizer step size:
  - Box-car pdf, error variance:
  - Max of N-Bit integer:

- SQNR:
Representative Circuit

Maximum signal power:

Total noise:

Peak SNR:
Peak SNR versus Capacitance

- SNR versus C for 1-V sinusoidal signal at 100°C

<table>
<thead>
<tr>
<th>Bits</th>
<th>SNR [dB]</th>
<th>C</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>20</td>
<td>4.1 aF</td>
<td>Difficult to make such small capacitors</td>
</tr>
<tr>
<td>6.3</td>
<td>40</td>
<td>412 aF</td>
<td>Matching considerations dominate</td>
</tr>
<tr>
<td>9.7</td>
<td>60</td>
<td>41 fF</td>
<td>Designer concerned about noise …</td>
</tr>
<tr>
<td>13.0</td>
<td>80</td>
<td>4.1 pF</td>
<td>… components often set by SNR</td>
</tr>
<tr>
<td>16.3</td>
<td>100</td>
<td>412 pF</td>
<td>Difficult battle with electronic noise …</td>
</tr>
<tr>
<td>19.6</td>
<td>120</td>
<td>41 nF</td>
<td></td>
</tr>
<tr>
<td>23.0</td>
<td>140</td>
<td>4.1 μF</td>
<td>Use a higher voltage …</td>
</tr>
</tbody>
</table>

- Achieving SNR > 100dB is extremely difficult
  - Rely on external components
  - Oversampling ADCs (see EE 240C)
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Noise Representations

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Noise Representations
Output and Input Referred Noise

Output

\[ v_{out} = H_i \cdot s_{n,i} \]

\[ \overline{v_{n,\text{out}}}^2 = \sum_i |H_i|^2 \overline{s_{n,i}^2} \]

Input

\[ v_{out} = H \cdot v_{in} \]

\[ \overline{v_{n,\text{in}}(\omega)} = \frac{\overline{v_{n,\text{out}}(\omega)}}{|H|^2} \]
Minimum Detectable Signal

\[ \overline{v_{i,eq}^2} \]

\[ V_s \]

noise model

noiseless amp

\[ v_o \]
Source Impedance

- Infinite source impedance $\rightarrow$ noise has nowhere to flow
  - Noiseless system???
Equivalent Input Current Noise
Input Voltage and Current Noise Sources
Correlated Noise
Examples

BJT Opamp

<table>
<thead>
<tr>
<th>LT1115 - Ultra-Low Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input Noise Voltage Density</strong></td>
</tr>
<tr>
<td>$f_0 = 10$Hz</td>
</tr>
<tr>
<td>$f_0 = 1000$Hz</td>
</tr>
<tr>
<td><strong>Wideband Noise</strong></td>
</tr>
<tr>
<td><strong>Corresponding Voltage Level</strong></td>
</tr>
<tr>
<td>re 0.775V</td>
</tr>
<tr>
<td><strong>Input Noise Current Density</strong> (Note 3)</td>
</tr>
<tr>
<td>$f_0 = 10$Hz</td>
</tr>
<tr>
<td>$f_0 = 1000$Hz</td>
</tr>
</tbody>
</table>

JFET Opamp

<table>
<thead>
<tr>
<th>OPA827</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input Voltage Noise:</strong></td>
</tr>
<tr>
<td>$f = 0.1$Hz to 10Hz</td>
</tr>
<tr>
<td><strong>Input Voltage Noise Density:</strong></td>
</tr>
<tr>
<td>$f = 1$kHz</td>
</tr>
<tr>
<td>$f = 10$kHz</td>
</tr>
<tr>
<td><strong>Input Current Noise Density:</strong></td>
</tr>
<tr>
<td>$f = 1$kHz</td>
</tr>
</tbody>
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Noise Calculations – Example

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Example: Negative Feedback Amplifier

\[ v_o = -v_i \frac{R_2}{R_1} = -a_v v_i \]

Noise Model

Opamp

Circuit

6 noise sources!
Noise Calculation

• One noise source at a time (linear superposition)

1) Noise at Output from $R_1$

\[ \overline{E}_1 = \overline{e}_1 \frac{R_2}{R_1} \]
\[ \overline{E}_1^2 = \overline{e}_1^2 \left( \frac{R_2}{R_1} \right)^2 \]
\[ \overline{e}_1^2 = \int 4kT R_1 \, df \]
2) Noise at Output from $R_2$

\[
E_2 = e_2 \\
E_2^2 = e_2^2 \\
e_2^2 = \int 4kTR_2 \, df
\]
3) Noise at Output from $R_3$

\[ E_1 = e_3 \left( \frac{R_1 + R_2}{R_1} \right) \]

\[ E_2^2 = e_2^2 \left( \frac{R_1 + R_2}{R_1} \right)^2 \]

\[ e_3^2 = \int 4kT R_3 \, df \]
Output Noise from Feedback Network
Input Referred Noise from Feedback Network
4) Noise from Opamp Voltage Noise

\[ E_{n}^2 = \int \left[ (en) \left( \frac{R_1 + R_2}{R_1} \right) \right]^2 df \]
5) Noise from Opamp Current Noise

\[ E_{np} = \int \left[ \left( i_{np} \right) \left( R_3 \left( \frac{R_1 + R_2}{R_1} \right) \right) \right]^2 df \]

\[ E_{nn} = \int \left[ \left( i_{nn} \right) \left( R_2 \right) \right]^2 df \]
Noise from Opamp
Total Input Referred Noise

\[ \overline{v_{i,eq}^2} = \overline{v_{R_1,n}^2 \left(1 + \frac{1}{a_v}\right)} + \overline{v_{v,n}^2 \left(1 + \frac{1}{a_v}\right)^2} + \overline{i_{i,n}^2 R_1^2} \]

- Source and feedback resistor
- Amplifier voltage noise
- Amplifier current noise

\[ \overline{v_{v,n}^2} = 4 \frac{nV}{\sqrt{Hz}} \quad \overline{i_{i,n}^2} = 1.2 \frac{pA}{\sqrt{Hz}} \quad \text{(uncorrelated)} \quad a_v \text{ large} \]

\[ R_1 = 50\Omega \quad \overline{v_{v,n}^2} \text{ dominates over } \overline{i_{i,n}^2}, \quad \text{correlation no concern} \]

\[ R_1 = 1M\Omega \]

\[ \frac{\overline{v_{i,eq}^2}}{\Delta f} = \sqrt{\left(0.9 \frac{nV}{\sqrt{Hz}}\right)^2 + \left(4 \frac{nV}{\sqrt{Hz}}\right)^2 + \left(0.06 \frac{nV}{\sqrt{Hz}}\right)^2} \]

Low source resistance: Voltage noise dominates
Use BJT

High source resistance: Current noise dominates
Use MOS
Additional Noise Topics

• Later in EE 240B
  – Noise in sampled data systems
  – (Low) noise amplifier design …

• RF noise metrics (EE 242A)
  – Noise figure
  – Receiver sensitivity
  – Phase noise in oscillators

• Cyclostationary noise
  – Noise in circuits with high signal amplitude which modulates the noise power spectral densities
  – E.g. oscillators, mixers, comparators
Summary

- Thermal noise is fundamental
- Random, but accurately described by universal statistics
- Strong correlation between noise and power dissipation for high accuracy analog systems
  - Up to 4x power for each extra bit
- Noise representations
  - PSD at output
  - Total noise at output
  - PSD at input (depends on $R_a$)
  - Minimum detectable signal (MDS)
- Noise contributions for different $R_a$
  - High $R_a$: current noise dominates (FET advantageous)
  - Low $R_a$: voltage noise dominates (BJT advantageous)