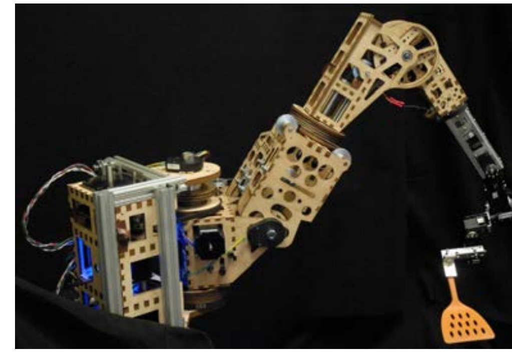


Sigma Hulls for Gaussian Belief Space Planning for Imprecise Articulated Robots amid Obstacles

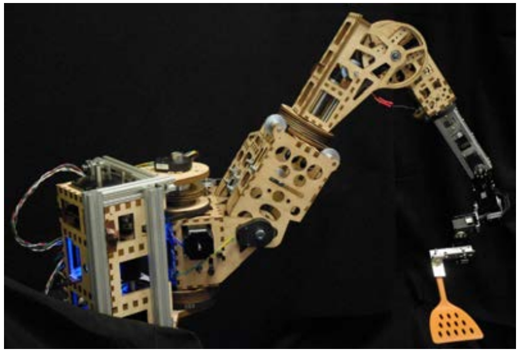
Alex Lee, Yan Duan, Sachin Patil, John Schulman, Zoe McCarthy, Jur van den Berg*, Ken Goldberg and Pieter Abbeel

UC Berkeley, *University of Utah

Motivation

Facilitate reliable operation of cost-effective robots that use:

- Imprecise actuation mechanisms – serial elastic actuators, cables
- Inaccurate sensors – encoders, gyros, accelerometers



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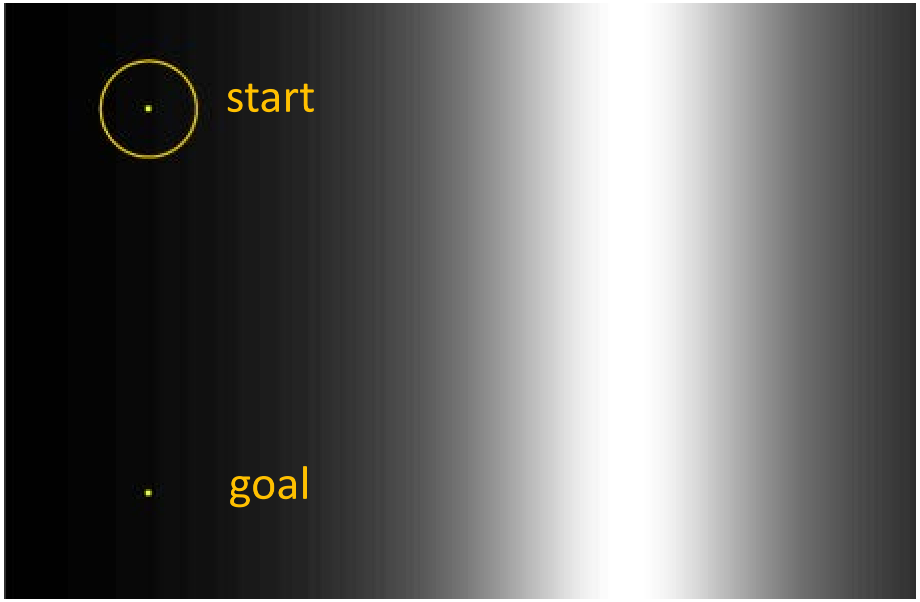
Prior Work on Gaussian Belief Space Planning

- Planning under motion and sensing uncertainty is a POMDP in general
  - Intractable in general
  - Compute locally optimal solutions
- Bry et al (ICRA 2011), Li et al (IJC 2007), van den Berg et al (IJRR 2011), van den Berg et al (IJRR 2012), Platt et al (RSS 2010)

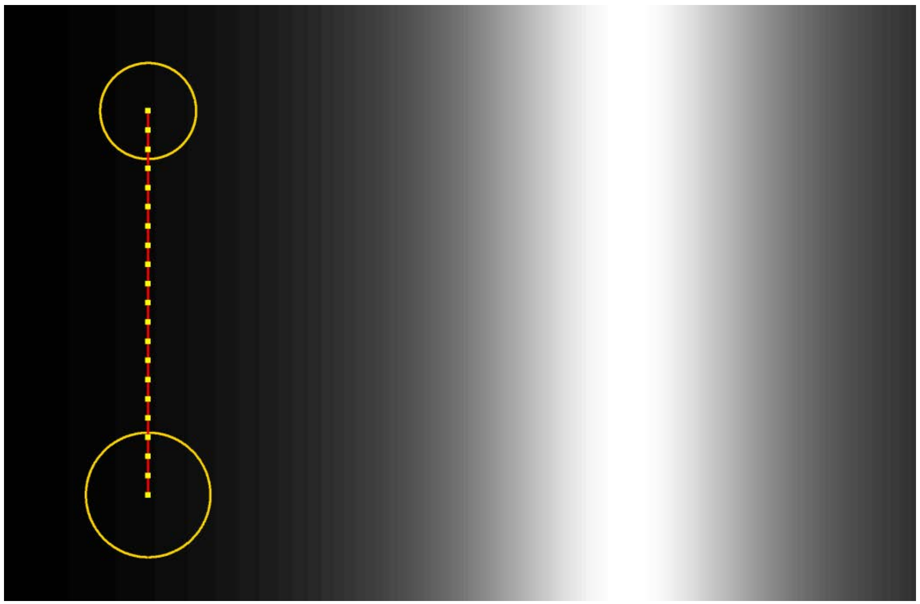
Gaussian Belief Space Planning



Problem Setup

[Example from Platt, Tedrake, Kaelbling, Lozano-Perez, 2010]

Gaussian Belief Space Planning



State space plan

[Example from Platt, Tedrake, Kaelbling, Lozano-Perez, 2010]

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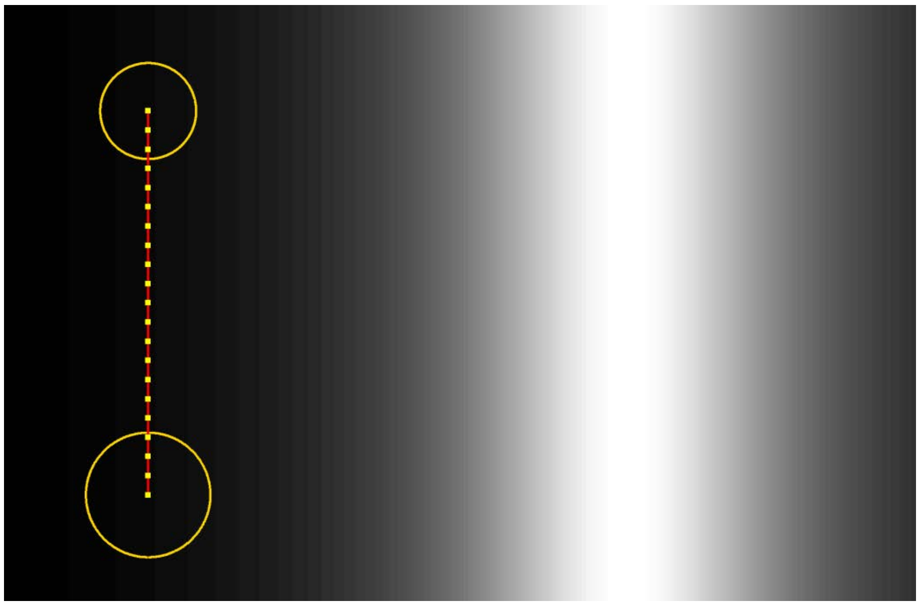
Gaussian Belief Space Planning



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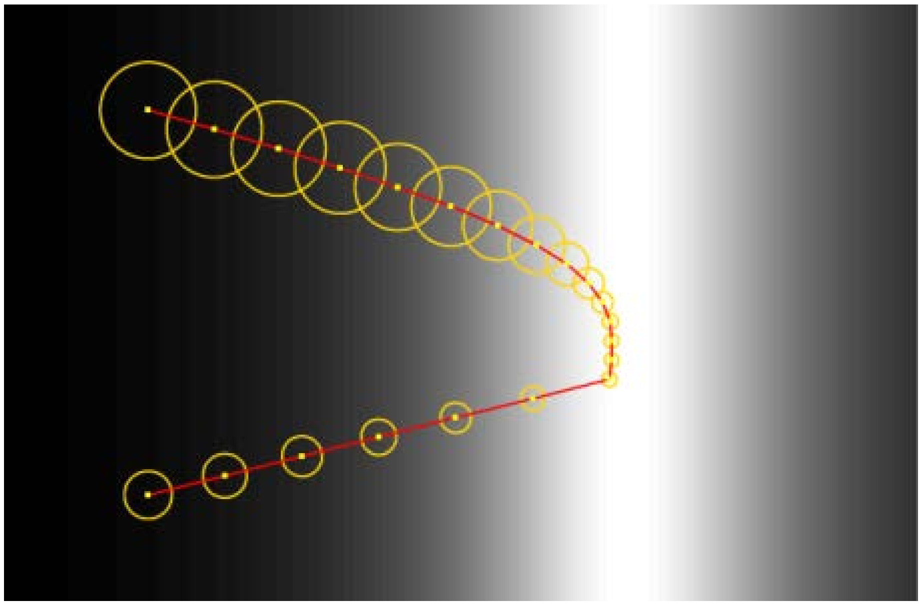

Belief space plan

[Example from Platt, Tedrake, Kaelbling, Lozano-Perez, 2010]

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Gaussian Belief Space Planning using Trajectory Optimization

- Gaussian belief state in joint space: $b_t = \begin{bmatrix} \mu_t \\ \sqrt{\Sigma_t} \end{bmatrix}$

  mean → μ_t

  square root of covariance → $\sqrt{\Sigma_t}$

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$$\min C(b_0, \dots, b_T, u_0, \dots, u_{T-1})$$
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- Non-convex optimization – Can be solved using sequential quadratic programming (SQP)

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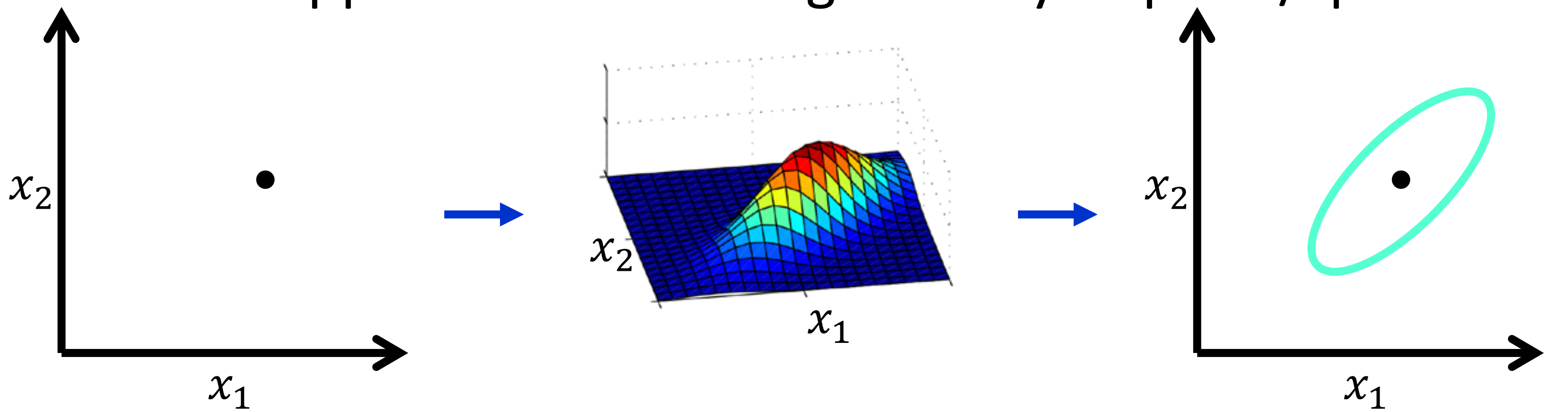

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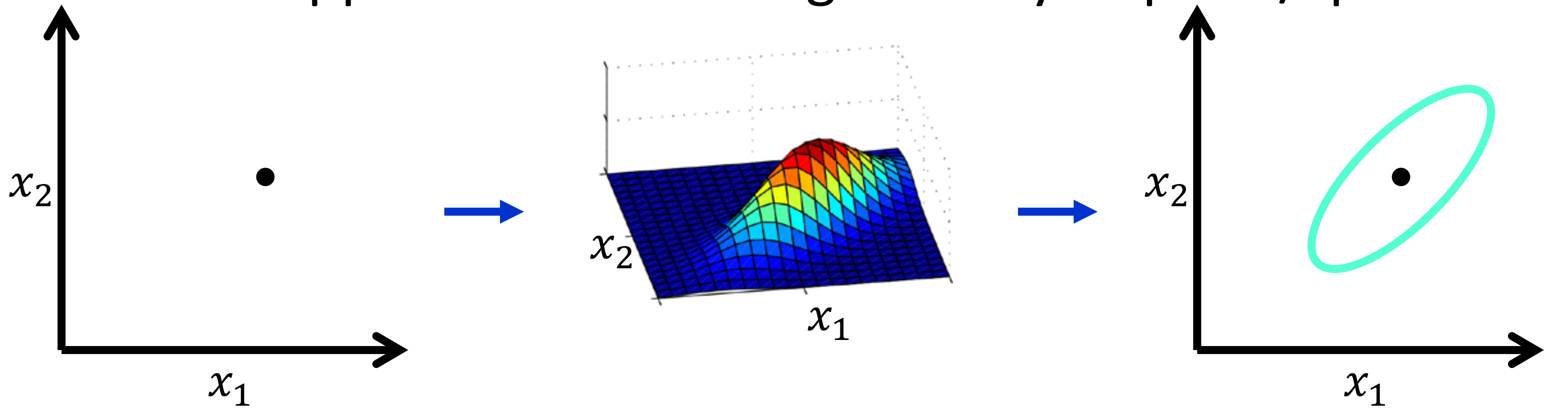
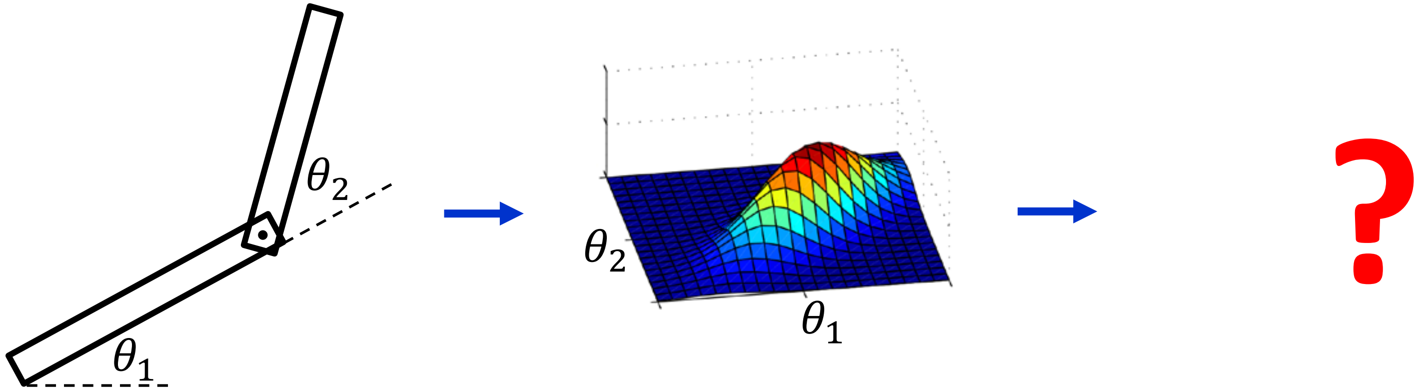


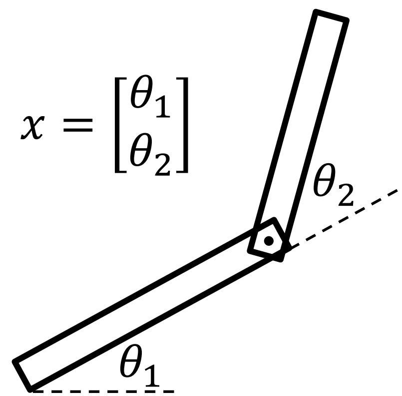
- How do you formulate the constraint for a robot link?

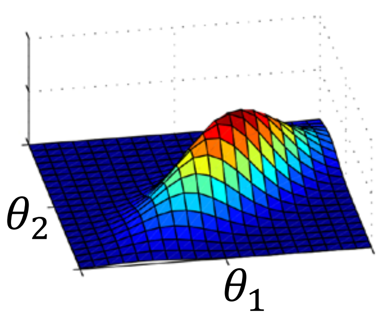


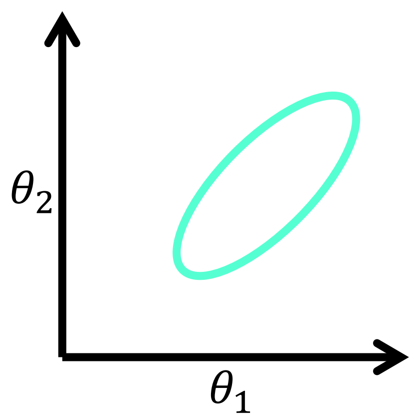
Presenter: Alex Lee (UC Berkeley)

Main Contribution: Incorporation of Collision Avoidance Constraints under Uncertainty through Sigma Hulls

$$x = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$







$$\mathcal{X} = [x \quad x \quad x \quad x \quad x] + \lambda[0 \quad \sqrt{\Sigma} \quad -\sqrt{\Sigma}]$$

Presenter: Alex Lee (UC Berkeley)

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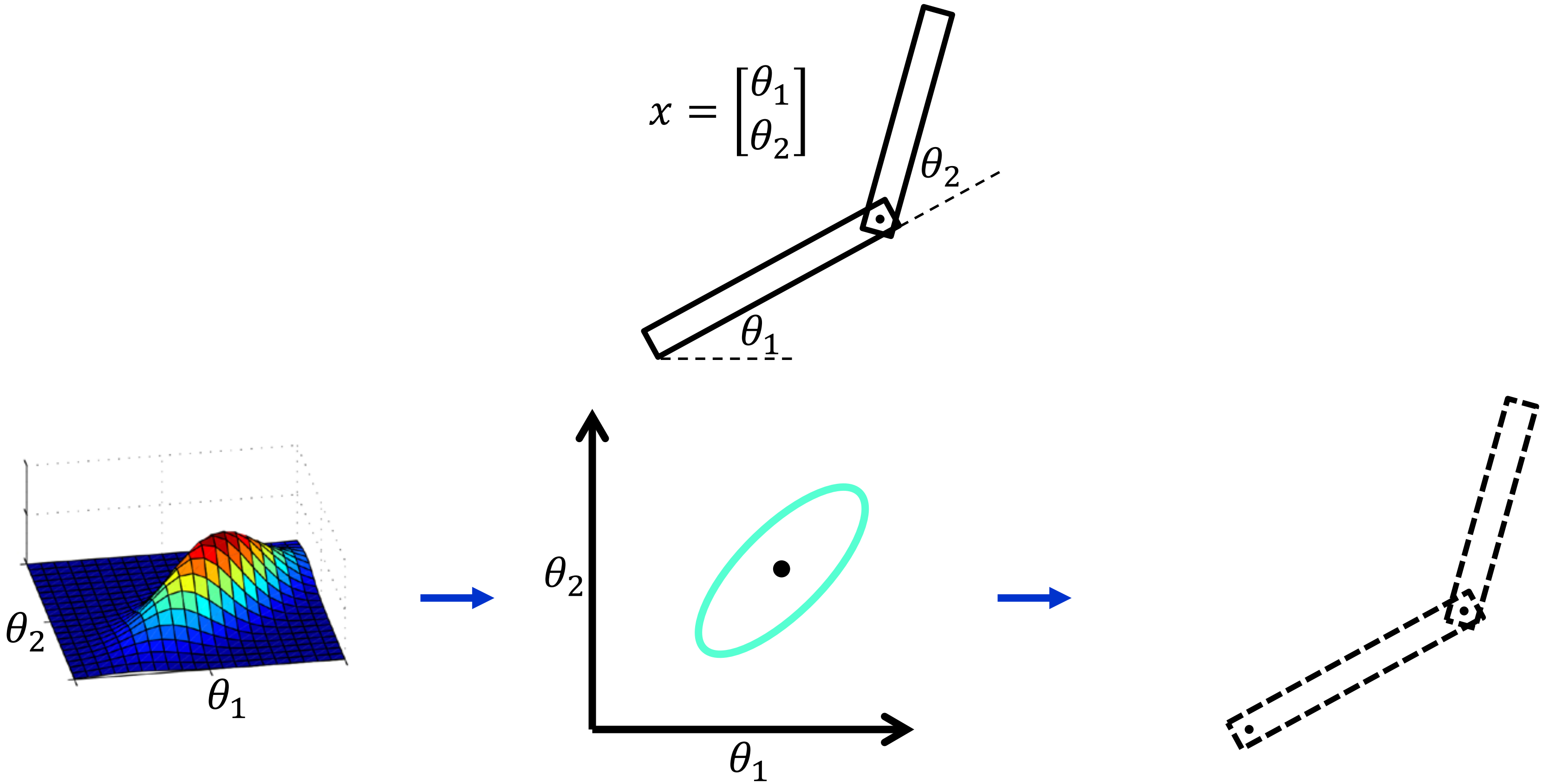

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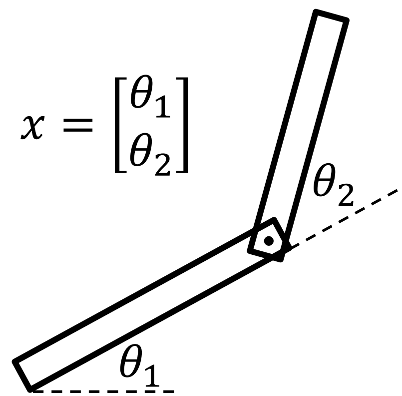

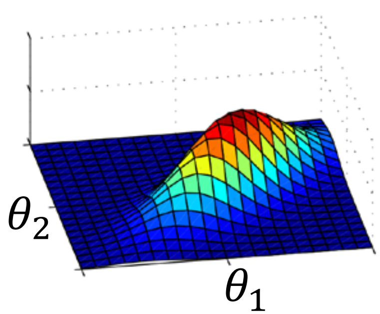

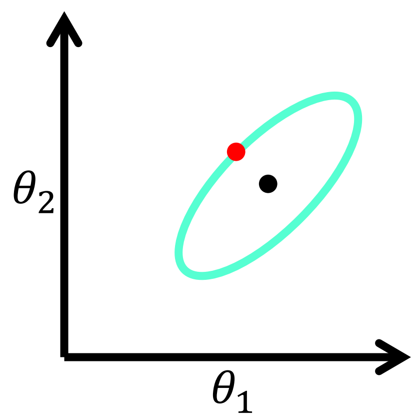

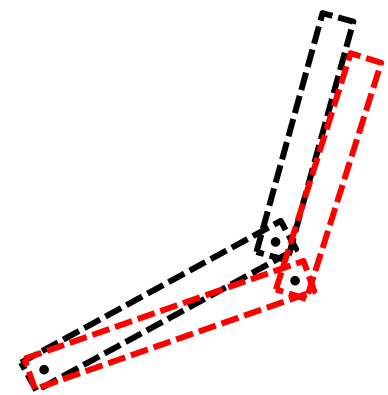

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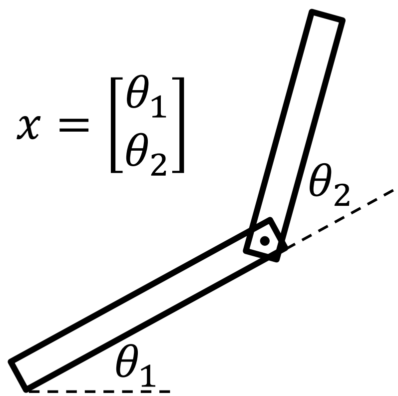

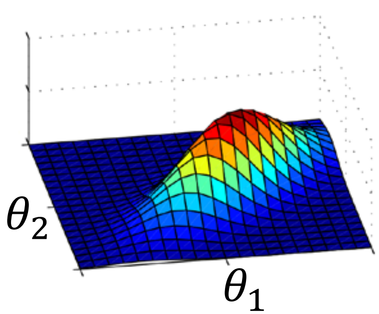

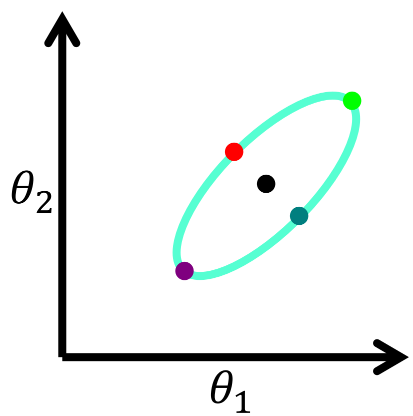

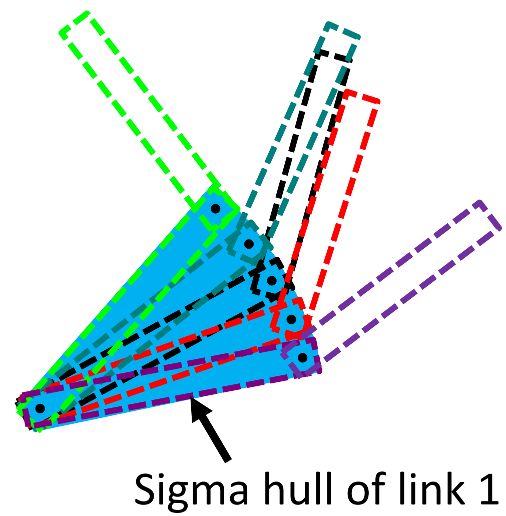

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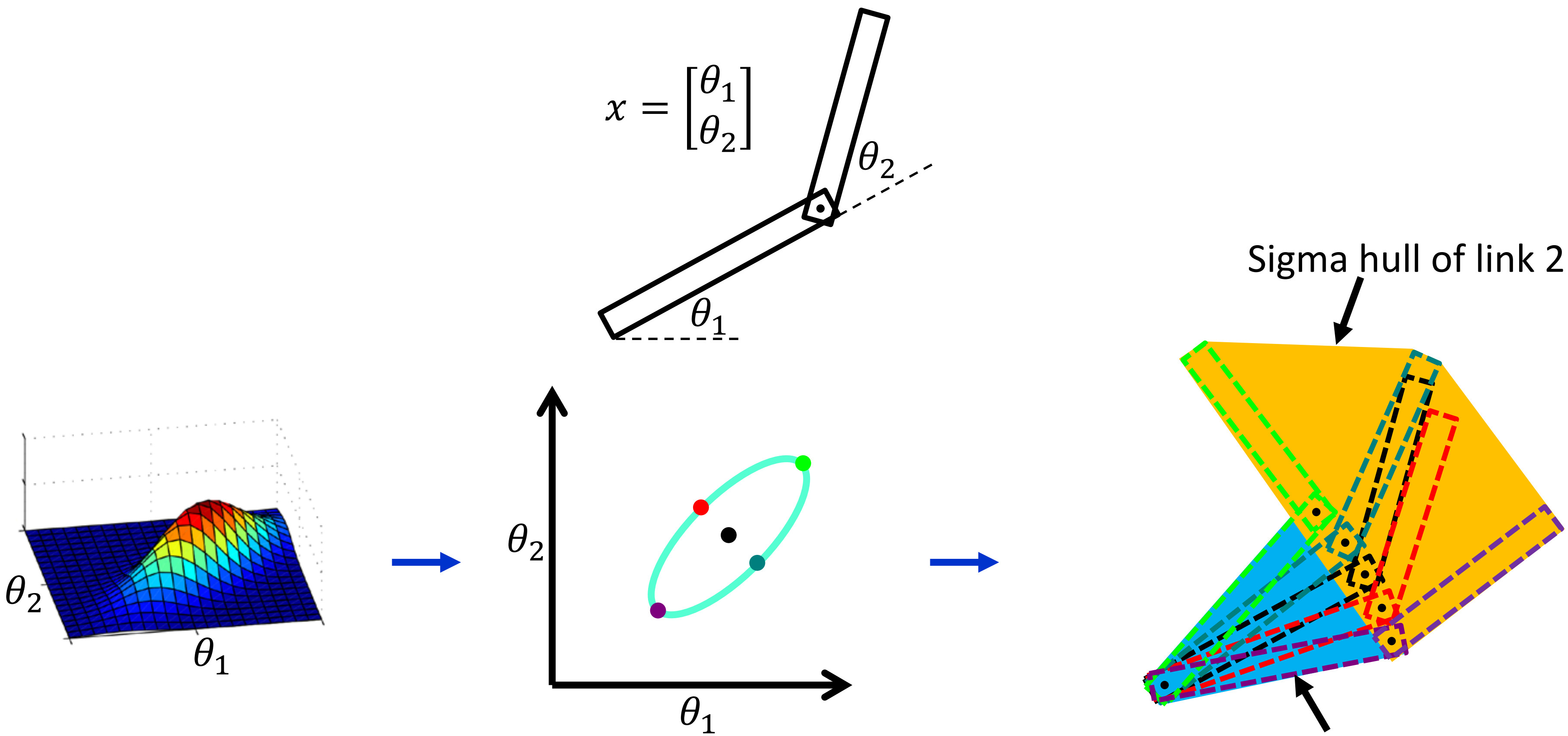
Main Contribution: Incorporation of Collision Avoidance Constraints under Uncertainty through Sigma Hulls

Sigma hull: Convex hull of a robot link transformed (in joint space) according to sigma points



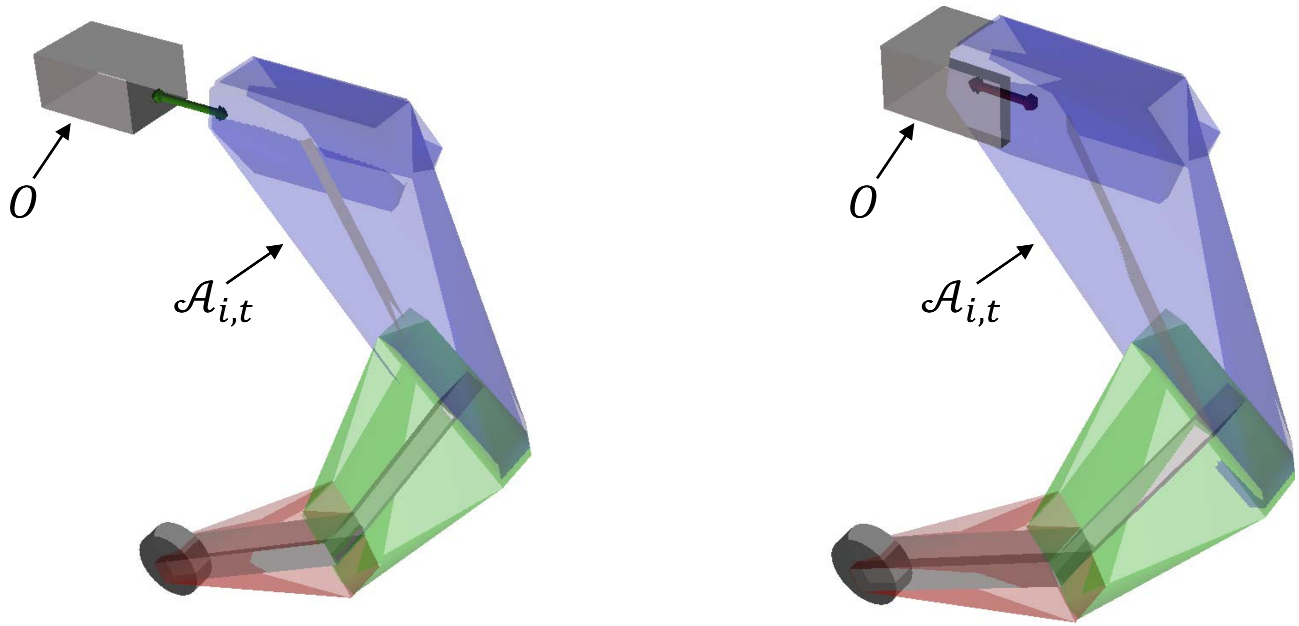
$$\mathcal{X} = [x \quad x \quad x \quad x \quad x] + \lambda[0 \quad \sqrt{\Sigma} \quad -\sqrt{\Sigma}]$$

Presenter: Alex Lee (UC Berkeley)

Signed Distance

Consider signed distance between obstacle O and sigma hull $\mathcal{A}_{i,t}$ of the i-th link at time t

$$\mathcal{A}_{i,t} = \text{sigmahull}(\text{link}_{i,t})$$



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- Use convex-convex collision detection (GJK and EPA algorithm)
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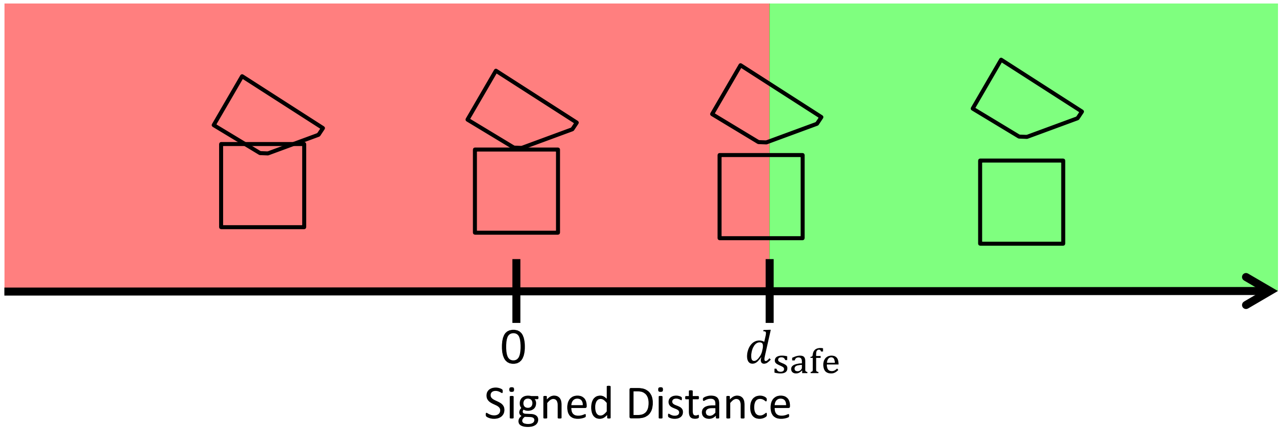
  $\forall$ times t, $\forall$ links i, $\forall$ obstacles O

$$\text{sd}\left(\mathcal{A}_{i,t}, O\right) \geq d_{\text{safe}}$$



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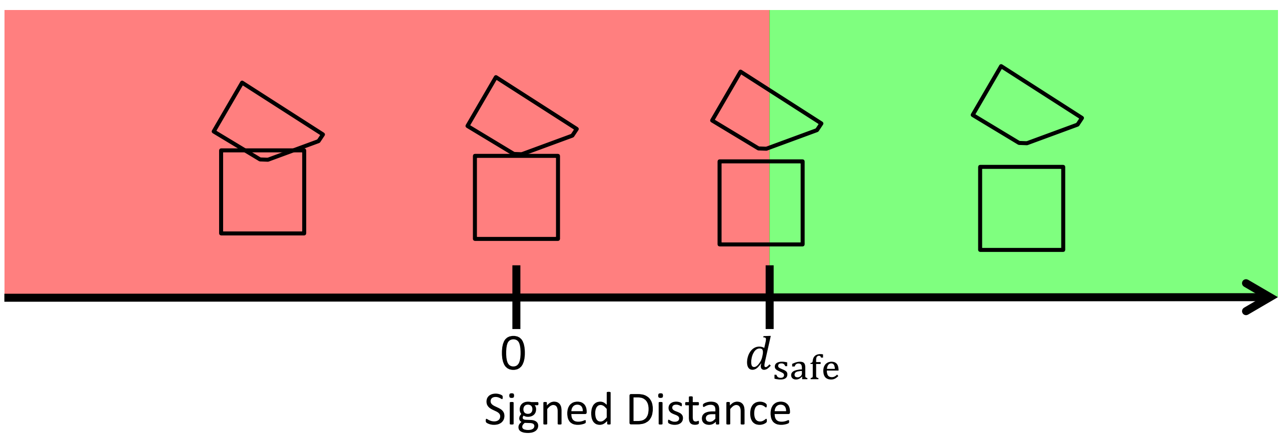
Non-convex!



- Use analytical gradients for the signed distance

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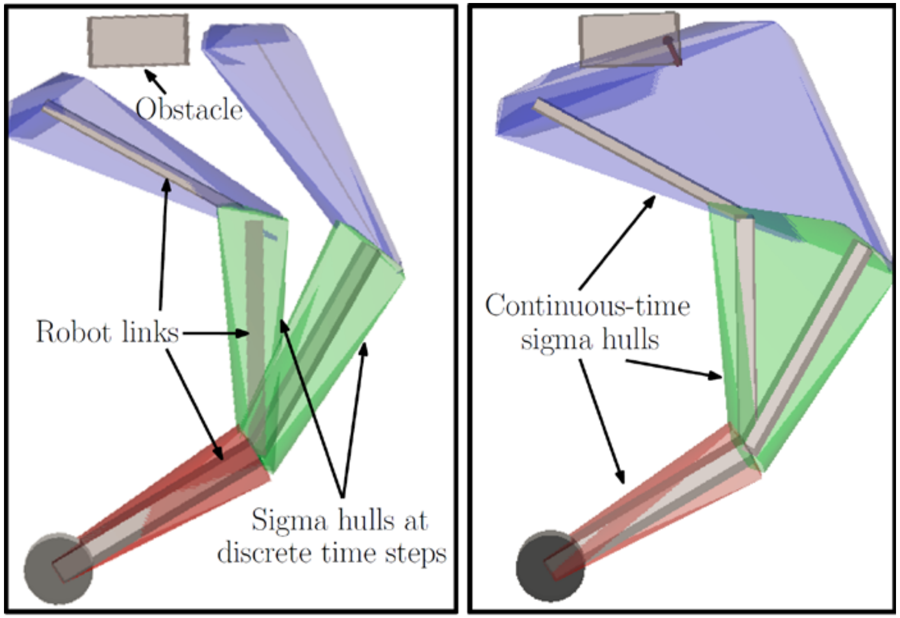
Continuous Collision Avoidance Constraint

- Discrete collision avoidance can lead to trajectories that collide with obstacles in between time steps



(a) Obstacle does not collide with discrete-time sigma hulls

(b) Obstacle overlaps with continuous-time sigma hulls

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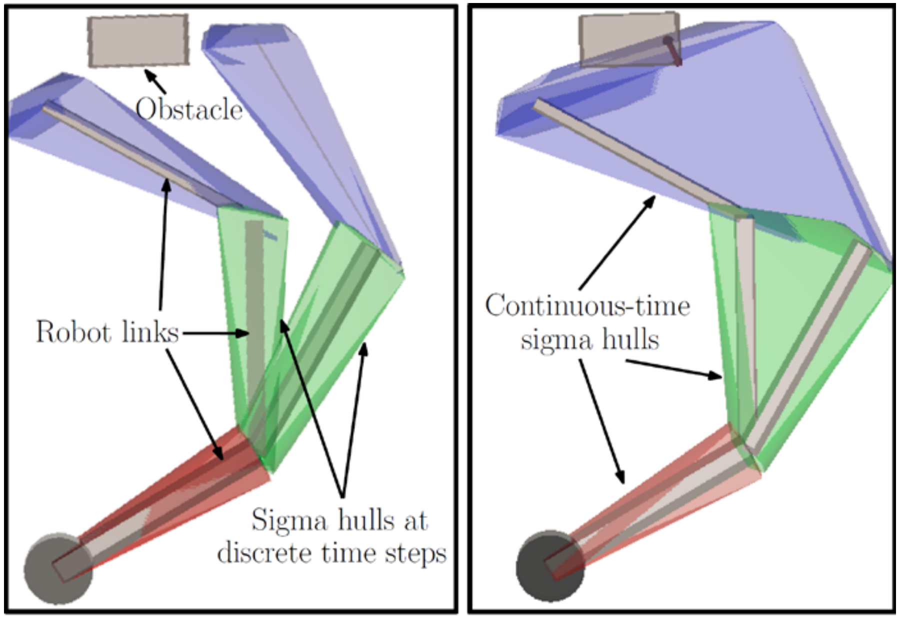
Continuous Collision Avoidance Constraint

- Discrete collision avoidance can lead to trajectories that collide with obstacles in between time steps
- Use convex hull of sigma hulls between consecutive time steps

$$\mathrm{sd}\big(\mathrm{convhull}(\mathcal{A}_{i,t}, \mathcal{A}_{i,t+1}), O\big) \geq d_{\mathrm{safe}} \qquad \forall\, t, i, O$$



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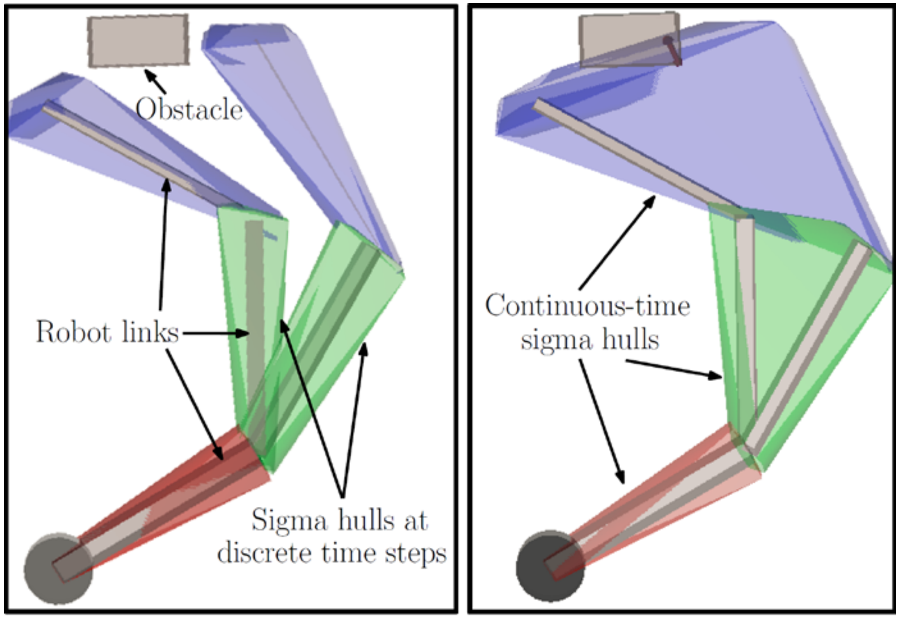
- Advantages:
  - Solutions are collision-free in between time-steps
  - Discretized trajectory can have less time-steps



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Gaussian Belief Space Planning using Trajectory Optimization

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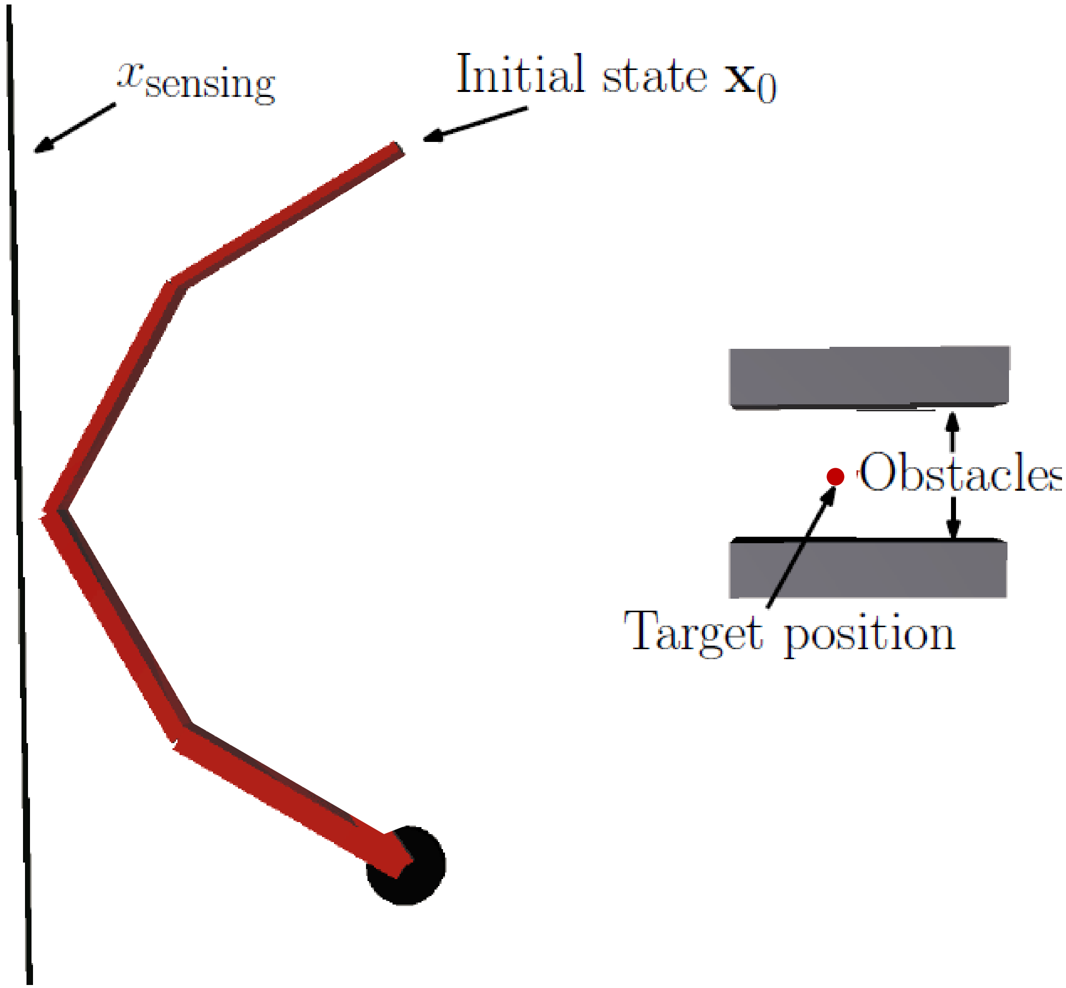
Model Predictive Control (MPC)

- During execution, re-plan after every belief state update
- Update the belief state based on the actual observation
- Effective feedback control, provided one can re-plan sufficiently fast

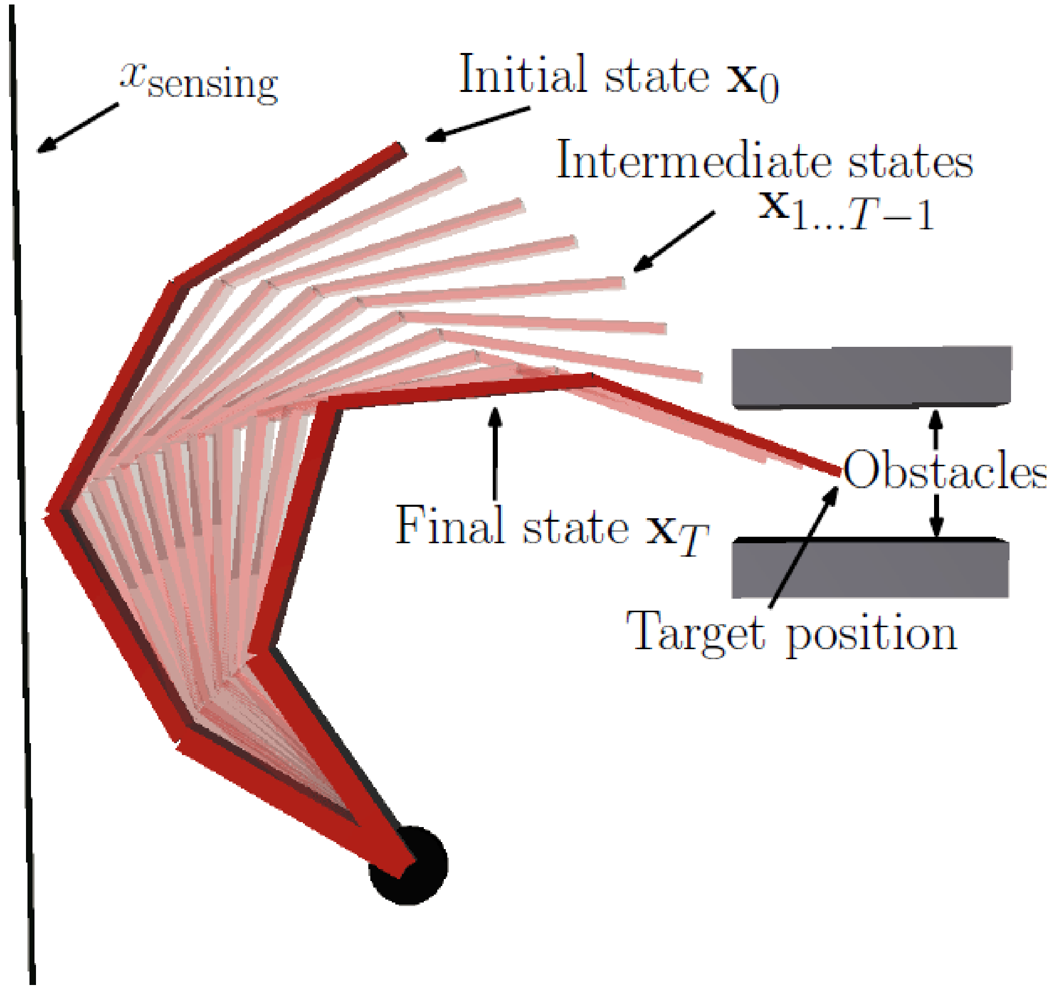
Example: 4-DOF planar robot

- Problem setup



Example: 4-DOF planar robot

State-space trajectory



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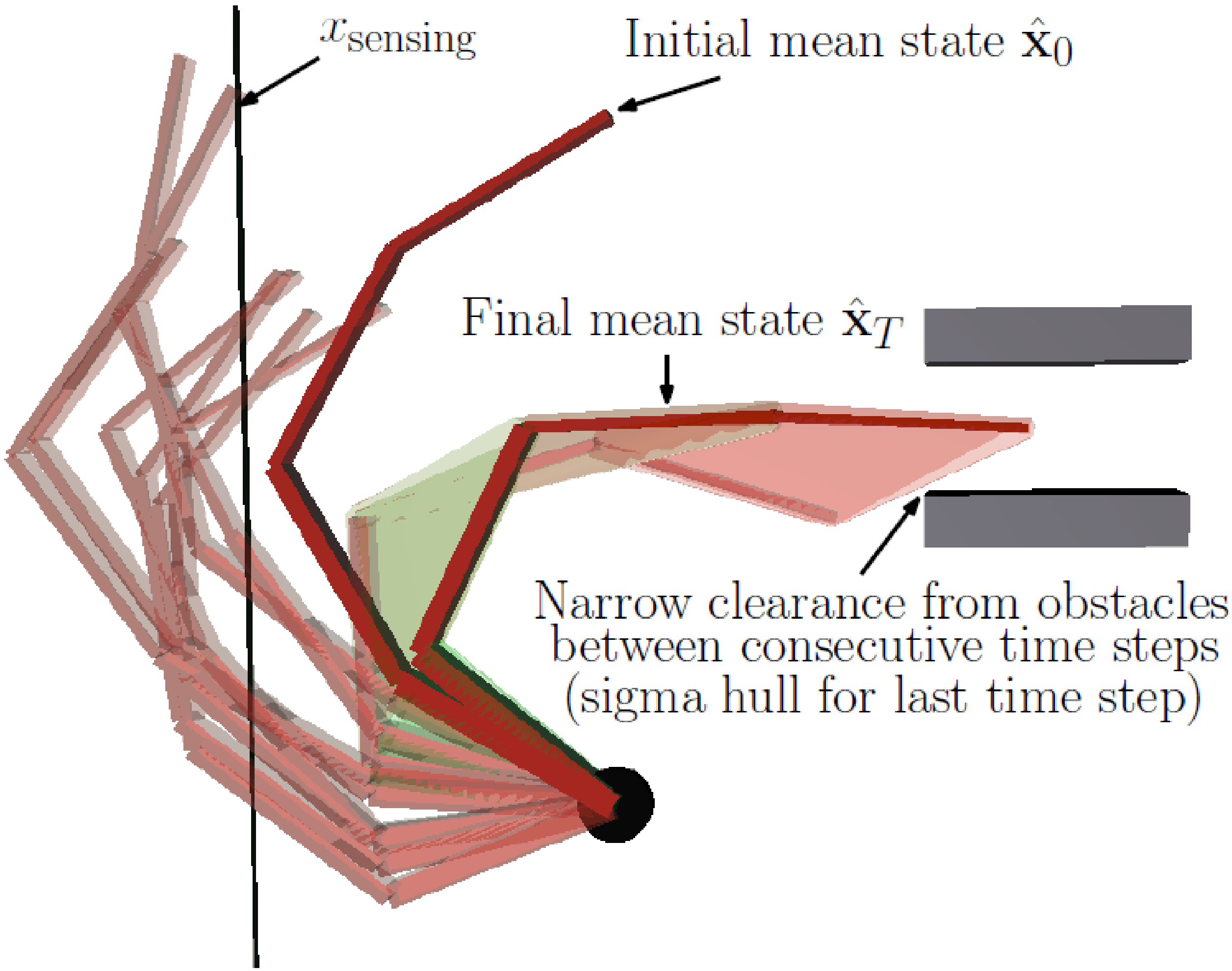
Example: 4-DOF planar robot

1-standard deviation belief space trajectory



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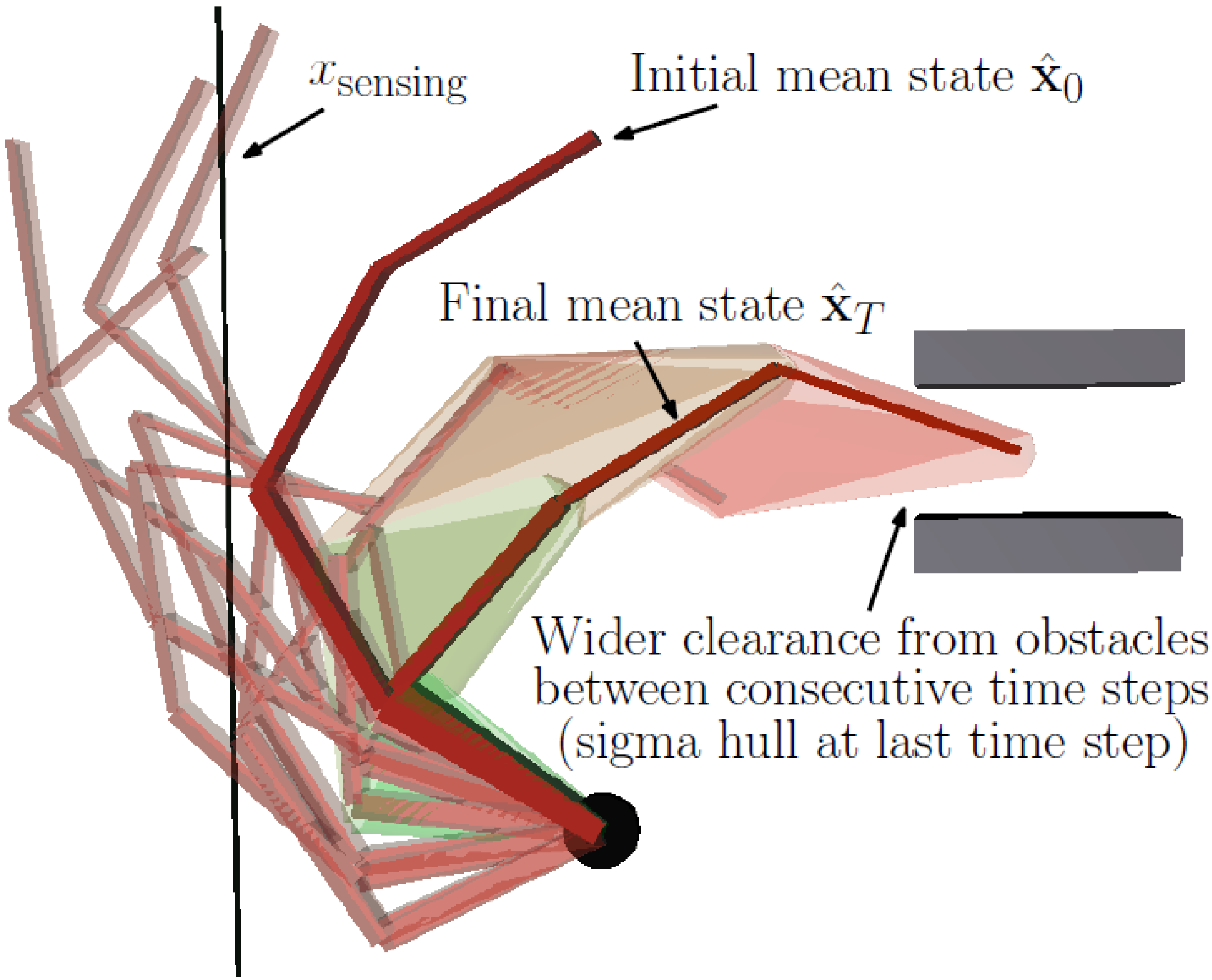
Example: 4-DOF planar robot

4-standard deviation belief space trajectory



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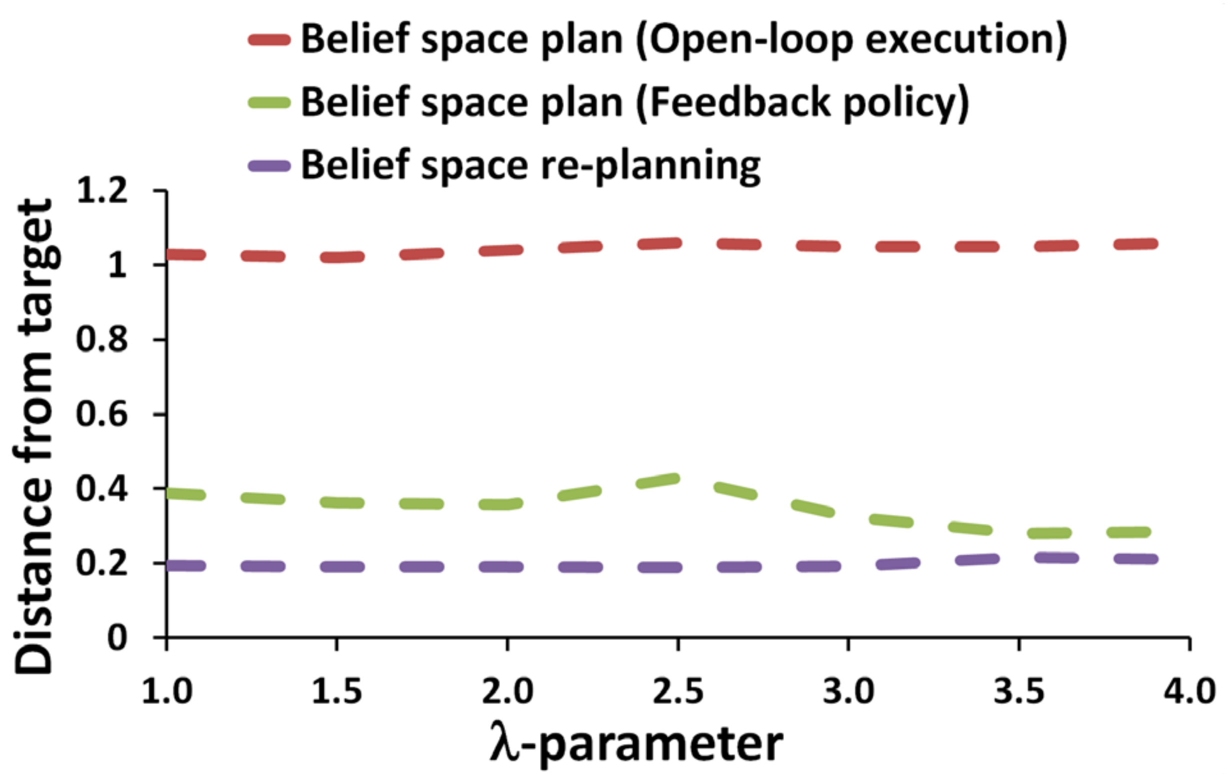
Experiments: 4-DOF planar robot

- Open-loop execution
- Feedback linear policy
- Re-planning (MPC)

Experiments: 4-DOF planar robot

Mean distance from target



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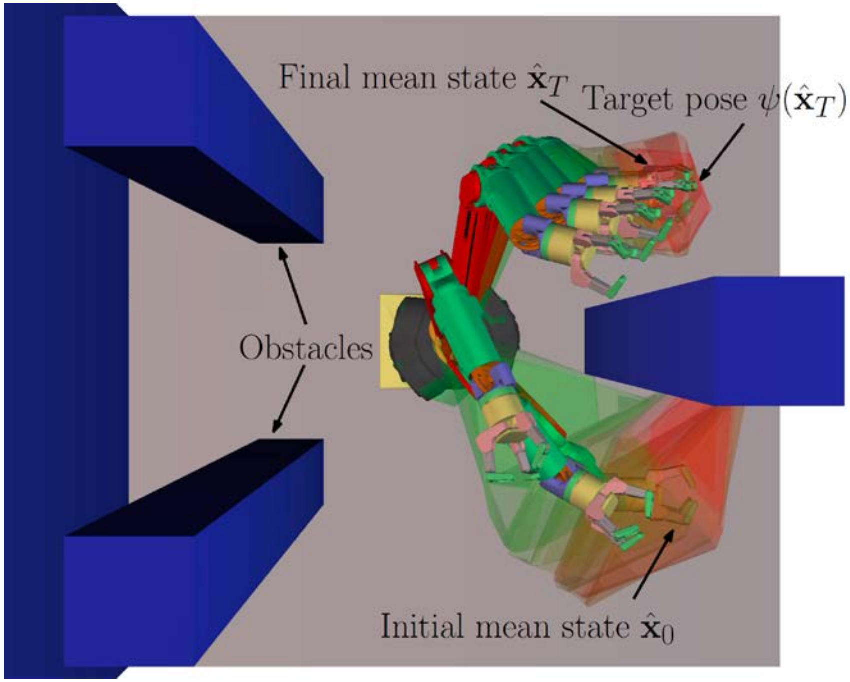
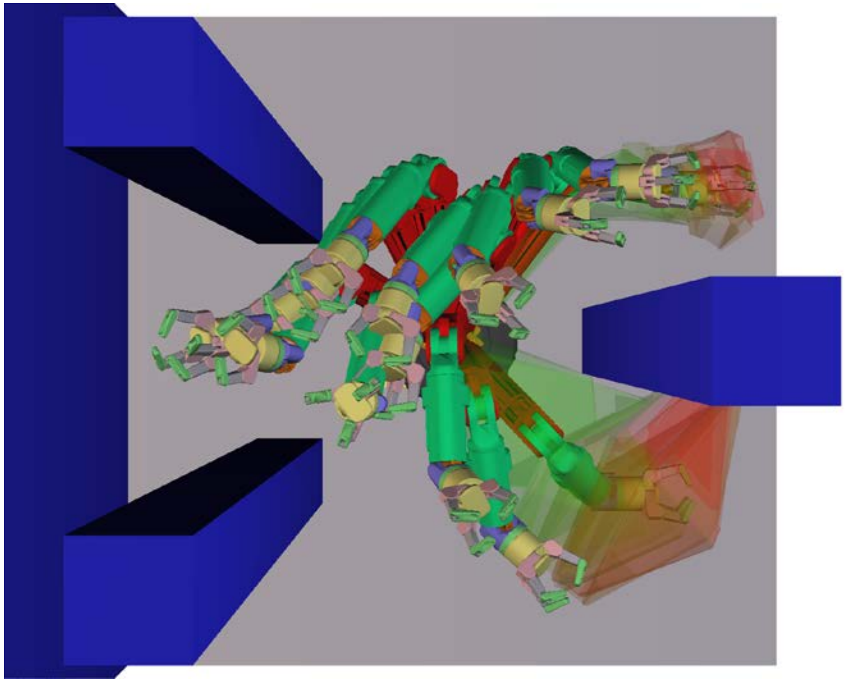
Example: 7-DOF articulated robot



State space trajectory

7 dimensions

1.9 secs



Belief space trajectory

35 dimensions

8.2 secs

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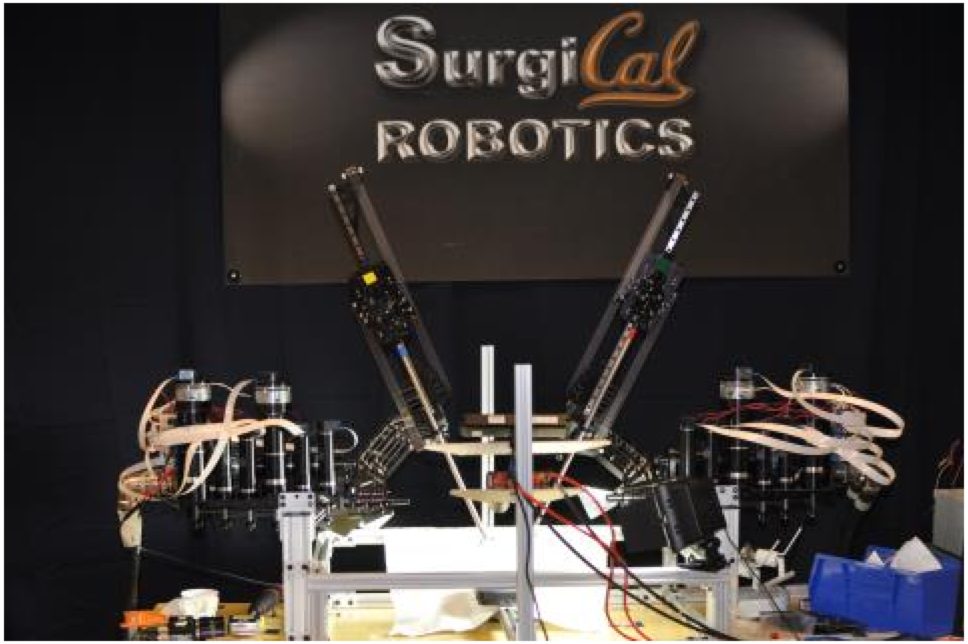
Extensions

- Planning in uncertain environments
- Multi-modal belief spaces
- Physical experiments with the Raven surgical robot



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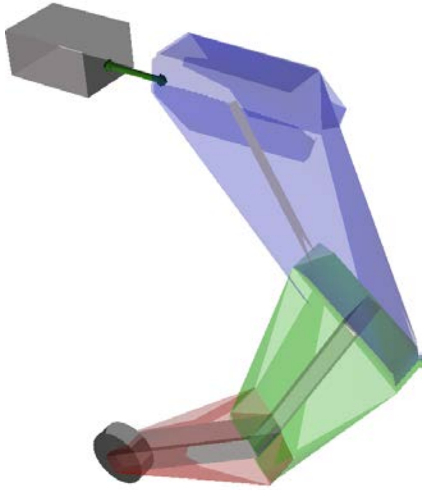
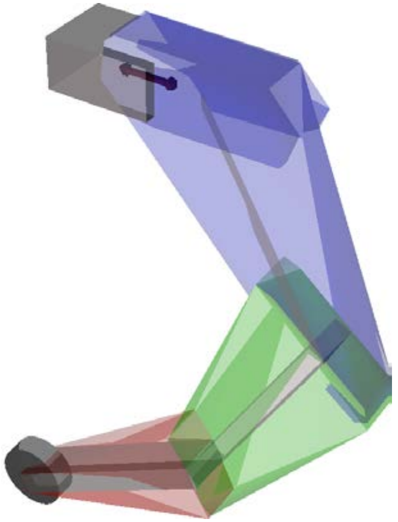
Conclusions

- Efficient trajectory optimization in Gaussian belief spaces to reduce task uncertainty
- Prior work approximates robot geometry as a point or a single sphere
- Pose collision constraints using signed distance between sigma hulls of robot links and obstacles
- Sigma hulls never explicitly computed – use fast convex collision detection and analytical gradients
- Iterative re-planning in belief space (MPC)





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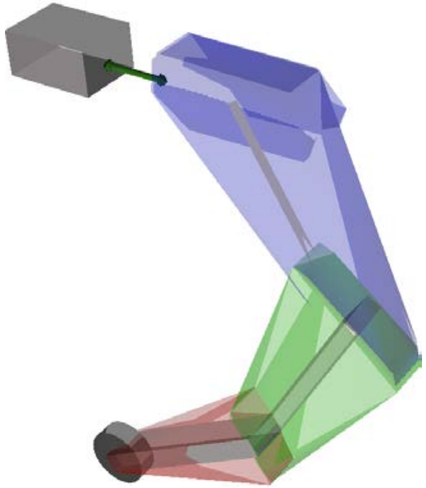
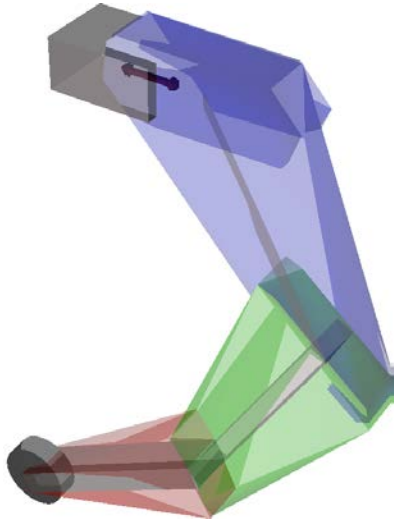
Thank You

- Code available upon request
- Contact: alexlee_gk@berkeley.edu





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